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CHEMICAL SECTION.

Stated Meeting, held Thursday, April 30, 1903.

Radio-Activity.

BY EDWARD A. PARTRIDGE and ROBERT H. BRADBURY.

I. HISTORICAL.

The history of science presents us with frequent illustrations of the fact that an erroneous hypothesis may be extremely helpful to the progress of knowledge. If an hypothesis is incorrect, investigation will reveal the error and the facts which are gathered in the course of the work will remain as a permanent heritage.

Radio-activity is a case in point. The investigations in this direction started from the idea that the Röntgen rays were due to the phosphorescence of the inner walls of the vacuum tube. It occurred to Becquerel that if this were true, then all phosphorescent substances should produce Röntgen rays when excited by light. To test this he wrapped a photographic plate in black paper, laid upon it a crystal of uranyl potassium sulphate—a highly phosphorescent substance—and exposed the whole to sunlight. The

plate was acted upon as he expected, but additional experiments revealed the surprising fact that an excitation of the uranium salt was unnecessary—the whole process occurred equally well in the dark. Further, it was found that uranium itself—which is not phosphorescent—exerted a more vigorous action on the plate than potassium uranyl sulphate.

At once it became necessary to drop the idea of rays excited by the action of light upon phosphorescent substances and to substitute the conception of uranium itself as a source of inexplicable radiations. All uranium compounds were found to be active, and there appeared to be a direct relation between the degree of activity and the uranium content.

At this stage (1898) P. and S. Curie began their remarkable work on the subject with a quantitative investigation of the radio-activity of a vast number of substances. Obviously the photographic method is unsuitable for quantitative work and can only give, at the best, rough approximations. Fortunately there is another effect of the Becquerel rays which can readily be measured—they confer upon air the power to conduct the electric current. If, then, two condenser plates are brought to a constant difference of potential and a radio-active substance is brought between them, a current will pass which can be measured. Other things being equal, the strength of this current will be proportional to the radio-activity of the substance. For rough work a charged electroscope is sufficient. The approach of a radio-active substance discharges it, and if the leaves are always brought to the same angle of divergence and the radio-active preparation held at the same distance, the time of discharge is an inverse measure of radio-activity.

Two of the results of this work of the Curies are especially important. First, compounds and minerals containing uranium and thorium are radio-active—in the absence of one of these elements radio-activity is not noticed. Second, some minerals containing uranium are far more active than uranium itself. Thus, chalcolite is about twice as active and some samples of pitchblende four times as active as metallic uranium.

Clearly it could no longer be held that the production of Becquerel rays is a property of the uranium atom, for in that case uranium would be the most active of all substances. Thus emerged the idea of elements of great radio-active energy accompanying uranium in its compounds. Varying radio-activity is easily accounted for by the supposition of a larger or smaller content of the real radio-active element. Guided by this thought, the Curies worked up large quantities of pitchblende by the ordinary analytical process, carefully testing each precipitate for radio-activity. The difficulty of the task was great, for pitchblende has an exceedingly complex composition, containing nearly all the known elements. The quantity of real radio-active substance in it is very small, probably one part in a million, so that it is necessary to work it up by the ton.

They were encouraged at the start by the observation that when the mineral is heated the sublimate obtained is much more active than the pitchblende itself.

II. PREPARATION AND PROPERTIES OF POLONIUM AND RADIUM.

In the course of this work the Curies found that the *bismuth* separated from the pitchblende was about 100 times as active as uranium. This led them to assume the presence in it of small quantities of a new radio-active element to which they gave the name of *polonium*. By various methods they were able to fractionate the precipitate and to increase its strength. But in all cases the radio-activity of the Curies' polonium preparations rapidly diminished with time and finally disappeared almost completely. This points to the conclusion that the Curies' polonium is simply bismuth which has been made secondarily radio-active by contact with the *radium* of the pitchblende.

Marckwald * has separated from the bismuth of pitchblende a radio-active substance which behaves in a very different way. He dissolved the active bismuth precipitate and subjected it to fractional electrolysis. The small quan-

* Berichte der deutschen chem. Gesellschaft, 35, p. 2285 ; 35, p. 4239.

tity of black substance first precipitated was intensely active. It appeared therefore that the radio-active element was a "nobler" metal—*i. e.*, had a smaller solution pressure—than bismuth itself. This led him to try the effect of placing a rod of bismuth in the solution. This was covered with a black coating and acquired an activity equal to that of the best radium preparations. The rays differ from those of radium in their complete lack of penetrating power. Wrapping the rod in filter-paper is sufficient to stop the radiation completely. This explains the fact that an increase in the thickness of the coating does not increase the strength of the radiation. The coating on one of Marckwald's rods weighed, when scraped off, only 5 milligrams. When it was heated a small quantity vaporized and the rest melted to a silvery globule which was extremely brittle. Since in the absence of concentration-differences in the solution a metal cannot precipitate itself, the new substance cannot be bismuth. Of late Marckwald has concluded that it exhibits the closest chemical similarity with tellurium, and proposes for it the provisional name *radio-active tellurium*. It is important to notice that the radiation of Marckwald's substance does not decrease with lapse of time. Bismuth rods coated in the way indicated are to be brought into commerce. They can be used for the purpose of distinguishing genuine diamonds, which phosphoresce on their approach, while imitation gems do not.

Let us now return to the work of the Curies. In the systematic separation of pitchblende they made the important observation that the *barium* obtained was strongly radio-active. The name *radium* has been given to the active element associated with the barium. In order to separate the two, various methods of fractionation must be resorted to. Thus radium chloride is less soluble in water and in very dilute hydrochloric acid. Giesel * finds that it is better to work with the bromides, since the solubility differences are greater. Radium has a higher atomic weight than barium, and Madame Curie followed the course of the

* Radio-active Substanzen und deren Strahlen. Ahrens' collection, vol. 7, p. 1. This article has furnished many facts for the present summary.

fractionation by determining the atomic weight, which steadily increased with the intensity of the radiation. In the following table the radio-activity is measured in terms of that of uranium as unity.

Radio-activity.	Atomic Weight.
60	138
3000	140
4700	140.9
7500	145.9
very great	175

She has obtained an atomic weight of 225 on a still purer preparation.

Giesel* kept track of the fractionation by means of the flame-color of radium, which is pure carmine.

The existence of radium is to be regarded as well established. The spectrum has been carefully examined by Demarçay, Giesel and Runge. Madame Curie has obtained practically pure $RaCl_2$ and Giesel, has obtained $RaBr_2$ in pure condition.

Radium bromide when kept in the air gives off bromine and acquires an alkaline reaction which is due to the formation of radium hydroxide. This absorbs CO_2 and passes into the carbonate. $RaBr_2$ is soluble in water, and the solution soon becomes yellow from the liberation of bromine. The crystals appear to contain an occluded gas—at least a gas escapes with effervescence when they are dissolved. A slow and apparently unlimited escape of gas takes place from the solution. The presence of a halogen somewhat complicates matters, but with non-halogen compounds the phenomenon is as simple as it is astounding. *The gas which is continuously liberated is a mixture of hydrogen and oxygen in the relation 2 : 1 by volume.*† There occurs, therefore, a kind of continuous electrolysis of the solution, but without any current or other assignable source of energy. The change is not a chemical reaction between the radium compound and the water, for already a quantity of hydrogen has escaped from Bodländer's solution, which corresponds to 10

* Berichte, 35, p. 3608 (1902).

† Bodländer. *Chemische Zeitschrift*, 2, p. 396 (1903).

atoms for every atom of radium it contains, and there are no signs of an end of the evolution. This simultaneous and separate escape of hydrogen and oxygen from the bosom of the same liquid gives a severe shock to certain old, cherished and well-grounded views about the activity of elements in the nascent state. Platinum, which exhibits a pretty general catalytic acceleration of reactions in which gases are formed and escape, retards this one since it brings about the reunion of the separated hydrogen and oxygen. Radium carbonate—an insoluble salt—produces a similar decomposition of water in which it is suspended. It has been remarked that a solution of radium bromide rapidly becomes yellow from separated bromine. The gas which is evolved from the liquid is a mixture of detonating gas with an excess of hydrogen equivalent to the bromine which has passed from the ionic to the molecular condition. This gas is itself radio-active, excites sensitive substances to phosphorescence and blackens the glass pipettes in which it is collected.

Radium compounds are luminous in the dark. One preparation containing radium and barium chlorides gave a light so bright that it was possible to read by it. The phosphorescence of pure radium compounds is bluish and intense, but soon fades. On the other hand, the impure preparations containing much barium retain their luminosity unchanged for years. When a radium preparation is brought near the closed eye there is a sensation of light which is probably due to the phosphorescence of the interior of the eye under the influence of the rays.

CHEMICAL AND PHYSIOLOGICAL EFFECTS OF BECQUEREL RAYS.

In Berthelot's* experiments on this subject the radium preparation was inclosed in a single or double-sealed tube so as to exclude the possibility of chemical action between it and the substance whose behavior was being investigated. Iodine pentoxide separated into iodine and oxygen. Nitric acid was colored yellow probably by a change of a similar

* *Compt. Rend.*, 133, p. 659.

character to its decomposition by light. However, in other cases the effects of the rays differed from those of light. Thus a solution of sulphur in carbon disulphide, which separates insoluble sulphur under the influence of light, remained unchanged by radium rays. Oxalic acid and acetylene, both of which are affected by light, were not acted upon. White phosphorus passes into red phosphorus under the influence of Becquerel rays. Oxygen is converted into ozone. Glass is colored brown or violet. Berthelot—who used a lead glass—considers that the change is due to the separation of metallic lead in the glass. Compounds of potassium and sodium are colored bluish-green or violet, the action being similar to that of Röntgen rays on the same substances except that it penetrates into the interior in place of being confined to the surface. The color fades when the substance is placed in the light. Elster and Geitel and Giesel believe that the colors are due to the separation of a small quantity of metal which forms a solid solution in the salt, and Giesel has produced similar effects by treating the haloid salts with the vapor of potassium or sodium. Paper becomes brown and brittle under Becquerel radiation and celluloid loses its solidity.

Many of the chemical effects just mentioned are endothermic and require a continuous supply of energy in order to bring the change about.

Inflammations on the fingers are a frequent sequence of work with radium preparations. Giesel placed a celluloid capsule containing a small quantity of a radium preparation against the skin of the arm for two hours. The result was a gradual but vigorous inflammation, followed by blistering and loss of the epidermis. Healing followed, but the growth of hair has not been renewed. The green parts of plants lose their chlorophyll under Becquerel radiation and turn brown or yellow. Seeds lose the power of germination.

The principal physical properties of the emanations from radio-active bodies are the following:

They render insulators temporarily conducting. A charged electroscope surrounded by air or paraffin is discharged in their presence.

Some of the rays are deflected by a magnetic field while others are not.

Solids are more or less transparent to them.

Many substances, such as barium platino-cyanide, fluoresce under the influence of the rays.

Radio-active substances continuously emit light and heat.

Some of the invisible rays are reflected according to the law for light.

According to Rutherford,* radium emits three kinds of rays which he denominates α, β and γ.

(1) The α-rays are readily stopped by thin layers of matter, even by tissue-paper, and produce the greatest part of the ionization of gases observed in ordinary experiments. They consist of positively charged particles.

(2) The β-rays consist of negatively charged particles and are similar in all respects to kathode rays. α and β-rays are deviated by a magnetic field.

(3) The γ-rays are not deviated by a magnetic field and are capable of great penetration.

The difference in penetrating power of the three classes of rays is shown by the following table, which gives the thickness of aluminium that reduces the intensity to one-half its original value:

α-rays, ·0005 centimeters.
β-rays, ·05 "
γ-rays, 8· "

The amount of energy given off by radium compounds as α-rays is about 1,000 times as great as that radiated as β-rays.

It has been established that the residual activity of uranium and thorium freed from uranium-x and thorium x consists entirely of α-rays. The radiation from uranium-x consists almost entirely of β-rays, while thorium-x sends out both α and β-rays. It appears, therefore, that the emission of α-rays is independent of the β-rays.

A thin-walled tube containing a radio-active substance

* *Phys. Zeit.*, 1903, p. 235

should show a diminution of weight in consequence of the continuous loss of changed particles. Heydweiller has shown that this is apparently the case. The loss of weight is, however, extremely small with the quantities he used, about ·0005 gram in four weeks. Rutherford points out that this may be due to the influence of the radiation upon the surface of the glass tube.

Curie and Laborde* have quantitatively examined the heat emitted by radium. One gram of radium gives off heat the amount of which is of the order of 100 gram calories per hour. One gram-atom of radium gives off per hour an amount of heat comparable with the heat produced by the combustion of a gram atom of hydrogen.

The measurements were made with a thermo-electric couple of iron and constantan, one junction of which was surrounded by radio-active $BaCl_2$ and the other by pure $BaCl_2$. The observations were confirmed by the Bunsen calorimeter.

Lodge† remarks that the development of heat is probably connected with the emission of the positively charged particles. He calculates that the emission of 1,000,000 charged atoms per second, with a velocity one-tenth that of light, would represent an amount of energy equal to 1,000 ergs per second, which would correspond to heat enough to melt 1 milligram of ice per hour. The loss of matter would amount to $\frac{1}{20}$ of a milligram in a century.

The position of radium in the periodic system is the subject of a recent paper by Runge and Precht.‡

As is well known, the spectra of related elements are definitely analogous. Runge and Precht found that the strongest lines of radium are exactly comparable to the strongest lines of barium and to the corresponding lines of magnesium, calcium and strontium. Moreover, the lines in these spectra occur in pairs, and the distances between the lines of the corresponding pairs are related to the atomic

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* *Electrician*, 1903, p. 970.

† *Nature*, 1903, p. 511.

‡ Runge and Precht: *Astrophysical Jour.*, 1903, p. 233.

weights of the elements to which they belong. The law is as follows: In every group of chemically related elements the atomic weights are proportional to some power of the distance between the lines of corresponding pairs. Usually the exponent of the power is not a whole number. This law can be transformed thus: "If the logarithms of the atomic weights and distance between lines of pairs be taken as co-ordinates the corresponding points of a group of chemically related elements will lie on a straight line."

This law is illustrated by *Fig. 1*, which is from Runge and Precht's paper. We see from this figure that, among the alkali metals, potassium is the only one that falls below the straight line through the remaining points. It is interesting to note that the law shows a comparatively large de

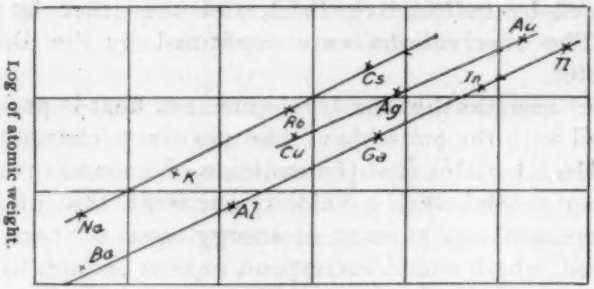

FIG. 1.

viation for potassium, since its atomic weight should be greater than that of argon to fit into the periodic system.

Fig. 2 shows the application of the law to the alkaline earth metals. Extrapolation gives 258 for the atomic weight of radium as opposed to 225, the value determined by Madame Curie. This would naturally be anticipated, since any impurity would lower the atomic weight, and it is very difficult to separate radium from barium. F. Rinne has found that the bromides of the two elements are isomorphous, so that it is probable that they would crystallize together in an isomorphous mixture. The number 225 fits the periodic system better in that it fills the gap between bismuth and thorium in the correct column.

For the atomic weight 258, radium would be displaced two rows farther in the column *Mg*, *Ca*, *Sr*, *Ba*, and a number of new unoccupied places would arise. The point remarked by Rutherford may be cited in support of the higher atomic weight. The greater the mass of the atom the more complicated is its probable structure, and electrons would be thrown off with greater ease.

Source of Energy.—Concerning the source of energy emitted by radio-active substances, we are absolutely in the dark. All that can be done is to mention some of the many hypotheses that have been offered to explain the phenomena.

(1) That the energy is chemical in character and is given off in consequence of a continuous modification of the *Ra*

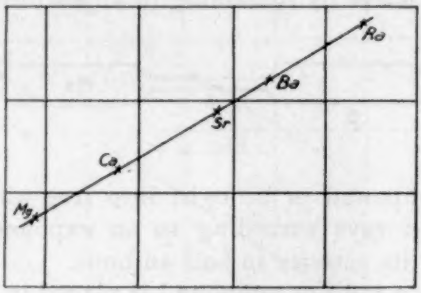

FIG. 2.

atom. The enormous amount given out renders this supposition highly improbable. Demarçay has advanced the argument, against this idea, that the spectrum of *Ra* shows no modification at the end of five months. Demarçay's notion does not seem valid, since to produce a spectrum energy is added in large amounts.

(2) That radio-active materials absorb energy of ether disturbances that do not noticeably affect ordinary substances.

(3) That atoms of *Ra* and similar substances act selectively upon the moving molecules of the surrounding gas after the manner of Maxwell's demons. (Crookes.)

(4) That radio-active substances absorb gravitational energy. To prove this some radium compound in a dish

was placed under one pan of a balance. That pan was apparently lighter than usual; the loss of weight gradually decreased. A second experimenter, however, observed that placing an empty dish under a balance pan causes apparent loss of weight, the necessary condition for the success of the experiment being that the dish must be touched with the fingers. Ascending currents of warm air completely account for the phenomenon. Repetition of the original experiment by Froch,* using great precautions, has shown that no loss of weight and hence no absorption of gravitational energy takes place.

Induced Radio-activity.—Substances that have been exposed to the emanations of radio-active bodies become themselves temporarily active. The law of the decrease of induced radio-activity has been carefully studied by Curie.† When a body made active by remaining in an enclosed space with

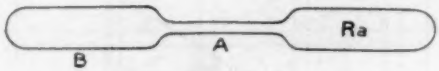

FIG. 3.

a radium compound is brought into free air it loses its power to emit rays according to an exponential law. It loses one-half its activity in half an hour.

Radium or a radium compound is placed in a tube of the form shown in *Fig. 3*. If after a time it is sealed off at *A* the portion *B* shows radio-activity, which decreases according to an exponential law, falling to one-half in four days. If *B* is opened and the enclosed air allowed to diffuse with the air of the room, the decrease of activity follows the rapid exponential law, that is, it falls to one-half in half an hour. If the tube was vacuous before sealing the rapid law was followed; this was also the case if air was admitted after sealing. The emanation apparently comes from the walls of the glass vessel, since immediately after removing the air the intensity is unaltered.

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* *Phys. Zeit.*, 1903, 318.

† *Phys. Zeit.*, 1903, p. 314.

The algebraic form of the law which the fall of intensity follows is:

$$I = I_0 e^{-\frac{t}{\theta}}$$

I = intensity at time t.
I_0 = initial intensity.
t = time since first observation.
θ = a constant time = 5·752 days.

Some observations were continued for twenty days, I had then fallen to $\frac{1}{27}$ its initial value.

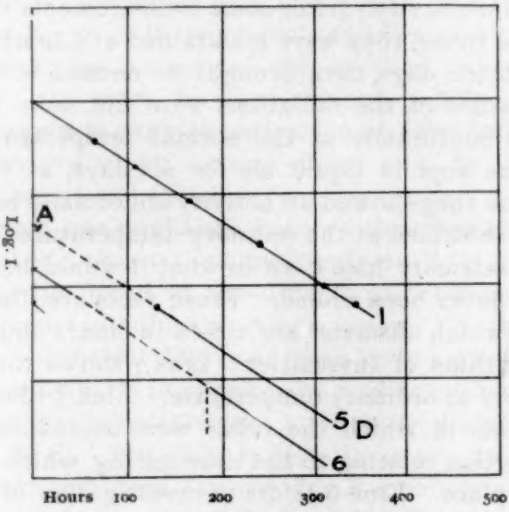

FIG. 4.

I falls to one-half its initial value in three days, twenty-three hours and forty-two minutes.

The constant θ remains the same under widely differing conditions. The following are some of the modifications of the experiment; in all cases θ was found to be the same:

(1) Radium solutions of very different activities were used.

(2) Solid Ra Cl_2.

(3) Form and dimensions of tube varied from 3 centimeters to 2,000 centimeters.

(4) Thickness of glass walls changed.

(5) Copper and aluminum vessels used in place of glass.

(6) Induction took place through tubes long and capillary, or short and wide.

(7) Time of induction varied between fifteen minutes and one month.

(8) Pressures varied from 1 atmosphere to 2 centimeters of mercury.

(9) Hydrogen or carbon dioxide used instead of air.

The same law is observed whether the external emanation or the internal conductivity of the tube is measured. Temperature has no influence upon the rate of loss of induced radio-activity. After some measurements were made with some tubes, they were maintained at a temperature of 450° for three days, then brought to normal temperature; the intensities of the radiations were the same as if they had been continually at the normal temperature. Other tubes were kept in liquid air for six days, at the end of which time they showed an activity abnormally low. After they had remained at the ordinary temperature for half an hour the intensity had risen to what it would have been if they had never been cooled. These facts are illustrated in *Fig. 4*, in which abscissæ are times in hours and ordinates are logarithms of intensities. Line 1 shows the decrease of intensity at ordinary temperature. Line 5 illustrates the experiments in which the tubes were heated to 450°, the dotted portion relating to the time during which the heating took place. Line 6 refers to investigation of the effect of refrigeration; the dotted line here shows time during which the tubes were cooled.

CENTRAL MANUAL TRAINING SCHOOL,
Philadelphia, April, 1903.

A GOOD ENGINE RECORD.

A straight line engine, in use at the works of the Solvay Process Company, in Syracuse, N. Y., recently completed a continuous run of 22 months, during which period it had not once been stopped. Its speed was 250 revolutions per minute, which makes 15,000 per hour, 360,000 per day, nearly 11,000,000 per month, and a grand total for the 22 months of some 241,000,000 revolutions without a stop. This speaks well for the excellence of the design and workmanship expended upon the engine, and for the excellence of the attendance received.—*Iron Age*.

PHYSICAL SECTION.

Stated Meeting, held Thursday, October 1, 1903.

Airy's Theory of the Rainbow.

By D. Hammer.

Up to some seventy years ago Descartes' view of the formation of the rainbow was universally accepted. In 1836 the English astronomer Airy published a new theory, but, as it often happens, it found few admirers, some admitting it only for the explanation of the so-called supernumerary bows. Of late years, however, several scholars have endeavored again to draw the attention of scientists to Airy's theory; as prominent among them may be mentioned Messrs. Mascart and Pernter.

Why, then, one might ask, is Airy's theory so little known whilst that of Descartes is almost universally taught? The reason may be found in the difficult calculations requisite for a full understanding of Airy's theory. If, therefore, it cannot, to a sufficient degree at least, be brought within the reach of non-experts in physics and higher mathematics, no one will require of our colleges and text-books to pay much attention to it. But this is not the case. In his "Ein Versuch, der richtigen Theorie des Regenbogens Eingang in die Mittelschulen zu verschaffen," Mr. Pernter succeeded in putting it into such form that it may without difficulty be introduced into high-schools and colleges.

In this essay we shall strive for the same end; our material is drawn from Mr. Pernter's writings on this subject.*

In order to show the differences between the two theories, we first give a short account of that of Descartes.

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* "Ein Versuch, der richtigen Theorie des Regenbogens Eingang in die Mittelschulen zu verschaffen," "Neues über den Regenbogen" and "Die Farben des Regenbogens."

Dr. J. M. Pernter is Professor at the University of Vienna (Austria); he is acknowledged as an authority in meteorology.

Fig. 1 represents the formation of the principal rainbow. A bundle of parallel rays strikes the raindrop. The one, which passes through the centre of the sphere, is not deviated from its course, but all the others undergo a change in their former direction, corresponding to the respective angles of incidence. Following up one of them we see that after two refractions and one reflection it makes a certain angle with the line *MN*, the axis of the rainbow. This angle we call *O*. If now we take different angles of incidence increasing at a constant rate from 1° to 90°, we find by construction as well as by calculation, that the angle *O* varies unequally, growing larger and larger, till, at an angle of incidence of about 59° 24′ having reached its maximum, it again retrogrades to 0. Near this limit *O* has its smallest rate of variation; hence much more light will be accumulated on this spot than at any other place of the illuminated field. It is this crowding together of (say) red rays that enables us to see the red of the rainbow. Now since every color has its own index of refraction every color has also its own maximum *O*. From this consideration it naturally follows that we must always see the well-known seven colors of the rainbow and that, provided the apparent diameter of the sun be the same, the width of the bow and its colors remain unchanged. The reason is, the index of refraction, on which the calculation depends, is a constant quantity, and the size of the raindrop, the only variable, is disregarded entirely. Hence the characteristic mark of Descartes' theory is constancy. It is simple in itself and easily understood, but facts prove its insufficiency.

To convince the reader of this assertion, we are to give an accurate description of the rainbow, a task which, though at first sight may seem superfluous, is of real necessity, if we wish to correct several misconceptions. "Want of accuracy in observing the phenomenon," says Mr. Pernter, "contributed perhaps more than anything else to the fact that many were satisfied with an inadequate explanation, thus keeping up until the present day a properly false theory which otherwise would have been dropped."

Here we need not dwell upon such particulars as the axis

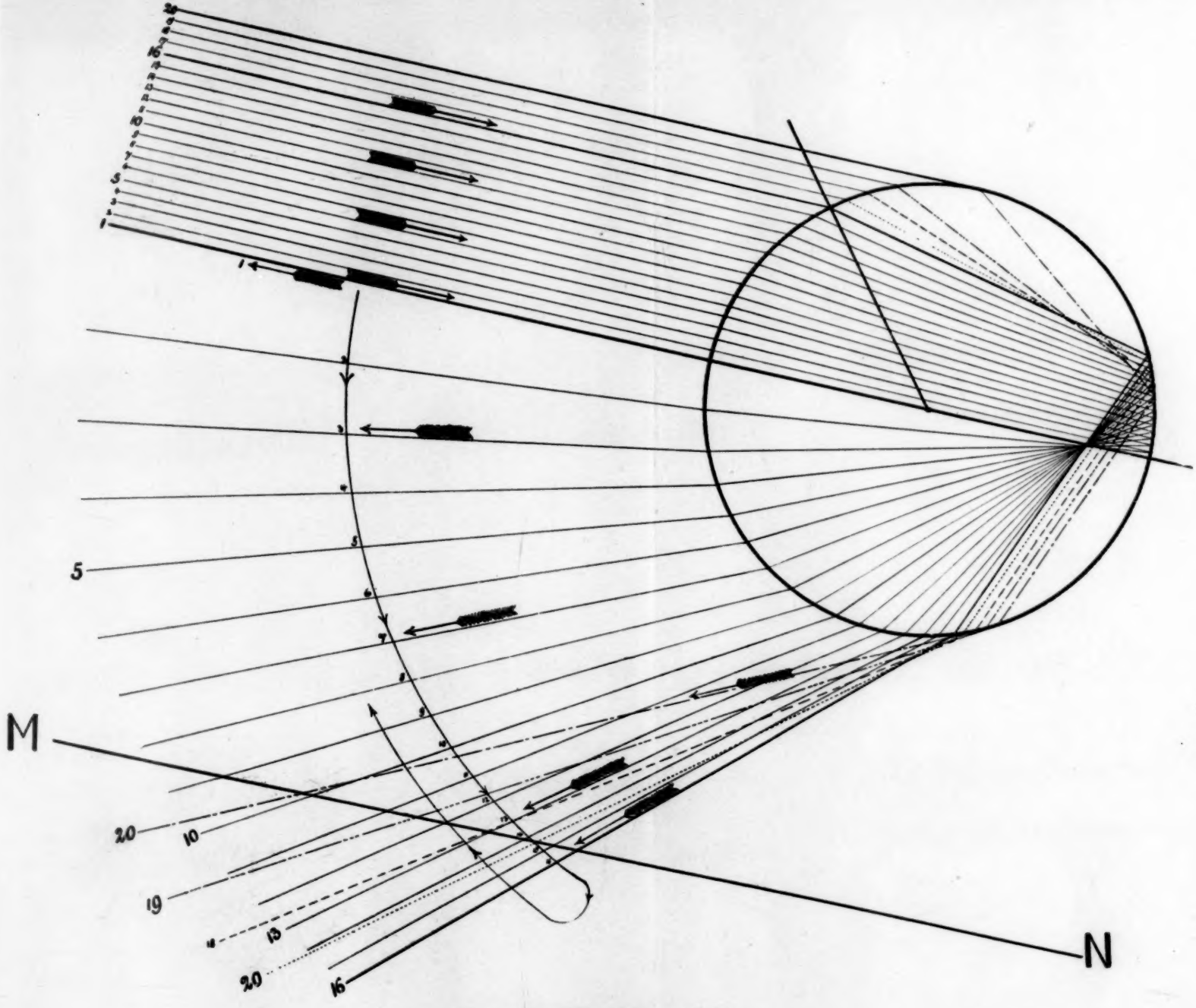


FIG. 1.—Illustrating Descartes' theory.

of the rainbow, the concentricity of the arches, their division into principal and supernumerary bows (in the sequel we will designate the former as first, second, third, etc., rainbows; the so-called supernumerary bows we comprise under the name of "secondary" bows). All this supposed as known, we briefly enumerate those features which are sometimes overlooked, although just they furnish a strong proof that Descartes' theory is far from being complete enough to give a sufficient explanation of the phenomenon.

(1) In the principal rainbows the colors are neither always the same nor is their distribution constant. Most frequently no blue is seen, in other cases a really pure red is wanting; dark blue occurs rather as an exception. Now, yellow almost disappears, whilst green and violet are prominent; again, yellow and green occupy the greater part, violet being reduced to a narrow extension. The intensity also changes: frequently the brightest spot lies in the beginning of violet. Finally, the variations in the width of both, arch and its colors, cannot escape the eye of any attentive observer.

(2) The secondary bows are characteristic of the rainbow, so much so that they are by no means a rare occurrence and their absence is rather the exception. They often lie hard against the violet of the first rainbow and extend inwards; under exceptionally favorable circumstances they border upon the violet of the second bow and extend outwards. They, too, in accordance with the entire phenomenon, undergo several changes: their number varies from six to one, sometimes they are wanting altogether; mostly made up of green and rose, they now and then show yellow, green and purple or yellow, green, blue and rose.

To the white rainbow we shall afterwards devote special attention on account of its importance for the new theory.

After this description of the rainbow, it is understood that that theory deserves our preference which accounts for all the numerous variations and explains the phenomenon in harmony with both light-theory and experiments. Airy's theory may justly claim this prerogative. According to it the rainbow is essentially a phenomenon of diffraction; this

statement, as we are going to show, derives its proofs from theory as well as experiments.

Fig. 2 is constructed for rays whose index of refraction (n) equals $\frac{4}{3}$ (red-orange near the Frauenhofer's line C). As is seen, the rays which after refractions and reflection are prolonged backwards, form each a certain angle with AS, the ray that passes through the centre of the sphere. This angle we call D. Its formula for all possible rainbows is:

$$D = k\pi + 2[i - (k + 1)r],$$

k standing for the number of reflections, i for the angle of incidence and r for that of refraction. Putting in p for $k + 1$, we have

$$D = k\pi + 2(i - pr).$$

Examining the figure more closely we find that the heavy line occupies a peculiar position; for the angle it makes with AS is a minimum. This angle, which we may denote by Dm, is to be found by means of the formulas:

$$\sin I = \sqrt{\frac{p^2 - n^2}{p^2 - 1}} \text{ and } \cos I = \sqrt{\frac{n^2 - 1}{p^2 - 1}},$$

I standing for the angle of incidence corresponding to Dm.

In the subjoined table Dm is worked out for the five first rainbows:

For Water.			For Glass.		
k.	I.	Dm.	k.	I.	Dm.
1	59° 24′	$\pi - 42° 4'$	1	49° 48′	$\pi - 22° 51'$
2	71° 49′	$\pi + 50° 56'$	2	66° 43′	$\pi + 86° 52'$
3	76° 50′	$2\pi - 42° 48'$	3	73° 13′	$2\pi + 9° 8'$
4	79° 38′	$2\pi + 43° 46'$	4	76° 48′	$3\pi - 71° 6'$
5	81° 20′	$3\pi - 51° 42'$	5	79° 6′	$3\pi + 27° 29'$

What then is the relation of the least-deviated ray to the formation of the rainbow? *Fig. 2* shows it. All the rays, which come from a single point of the sun's surface and strike the drop, are practically parallel, owing to the great distance of the sun. Hence the wave-front can be desig-

nated by the line *AB*, perpendicular to one of them. But when the same rays leave the drop, their wave-surface has necessarily become curved as shown in the figure by *ab*. The curvature is such that the least-deviated ray forms its turning point. But somewhat divergent rays cannot be united into one image in our eyes, hence, in case the rainbow is perceived, it can only be ascribed to rays that deviate least from parallelism: such are those in the nearest vicinity of the least-deviated ray. The very small portion of the wave, the rays of which are effective, is represented in *Fig. 2a*, being, of course, highly magnified.

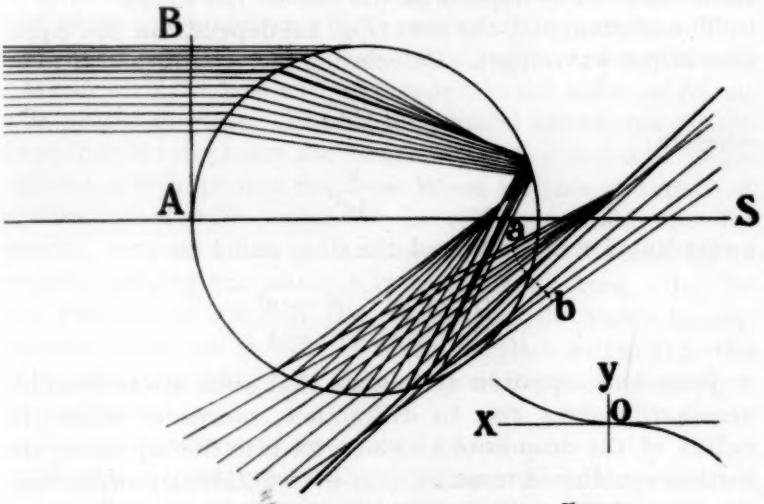


FIG. 2a.

FIG. 2.—Illustrating Airy's theory.

The mere shape of this wave-front may tell us that we have to deal with the phenomenon of diffraction. This statement is based on Huyghen's principle, which says that the effect of a wave surface must be considered as the sum total of the effects of each point in the surface, whereby each point is taken as a wave-former of its own, *i. e*, it sends out rays in all directions. In case of spherical wave-fronts the final result is the same as if the wave had been produced only in the centre of the sphere. But this is not true for other curved waves, particularly if there is question of cur-

vatures having a turning point. In this case diffraction must take place. If we use monochromatic light, an alternate series of light and dark bands will be observed; with white light these bands show a most beautiful variation of hues. Hence the phenomenon is analogous to that observed through a diffraction-grating; still it offers a somewhat different character, since the producing wave-surface is of a peculiar shape.

We now come to a point which Descartes' theory does not at all take into account, namely, the fact that the ever varying distribution of colors and consequently the rainbow itself necessarily depend on the size of the drops.

The efficiency of the rays (*Fig. 2a*) depends on the equation of the wave-front. Calculation has established it to be

$$y = Hx^3,$$

wherein

$$H = \frac{h}{3a^2},$$

a standing for the radius of the drop and h for

$$\frac{(p^2 - 1)^2}{p^2(n^2 - 1)} \sqrt{\frac{p^2 - n^2}{n^2 - 1}}.$$

From this equation it follows that such a wave-surface necessarily gives rise to diffraction; moreover, since the radius of the drop enters as a factor into the equation, the further conclusion must be, that with differently sized drops the phenomenon will show a different character. Given the equation and the radius of the drop we are enabled beforehand to construct the rainbow that will result. Mr. Pernter undertook this weary task and in an essay on the colors of the rainbow published the results obtained for drops whose radius respectively is: 1,000, 500, 250, 150, 100, 50, 40, 30, 25, 20, 15, 10, 5 μ ($\mu = \frac{1}{1000}$ millimeters.)

The following four diagrams we present to the reader remarking that they are not the result of mere speculation, but that, partly at least, they were proved correct by actual experiments.

We must now prove that these theoretical conclusions

are borne out by experiments, which, in order to show the principal features of the phenomenon, may be performed without great difficulty.

A few remarks must be made regarding the mode of procedure.

Since the right section of a cylinder is a circle as well as the section of a sphere, we may in carrying out the experiment make use of cylinders or of spheres indifferently; the only difference will be that with cylinders we shall obtain a straight band and with spheres a real bow, a circumstance of little importance. It is obvious, moreover, that essentially the same phenomenon will be obtained in case the light falls upon glass instead of water; the angles under which we have to view the bow will correspond to the index of refraction proper to glass. The latter method affords the advantage that it brings out the image distinct, sharp and richly colored, which is not the case when we use cylinders or spheres filled with water. It is self-understood that for making very accurate researches it would be necessary to employ nothing but water, a rather difficult task. But for the purpose of proving the correctness of Airy's theory, cylinders, spheres or even threads of glass will suffice, the only condition being that they are well formed and without impurities.

In the experiments we proceed as follows:

We use a spectrometer the telescope of which combines a large field of view with a moderate magnifying power. In the centre of the apparatus we attach a glass cylinder from 2 to 3 millimeters in diameter and put a red glass in front of the slit. Then through a porte-lumière we allow a beam of sunlight to pass to the cylinder. Since in the first bow (see table above) the light is deviated from its original direction by an angle of $\pi - 22° 51'$, the telescope must be turned accordingly. In this position we see a long series of red bands growing less in intensity and width. Next we put in, instead of the red glass, a blue one: only blue bands are observed, with the difference, however, that the entire spectrum is somewhat displaced. These observations, therefore, give an experimental proof for the assertion that we

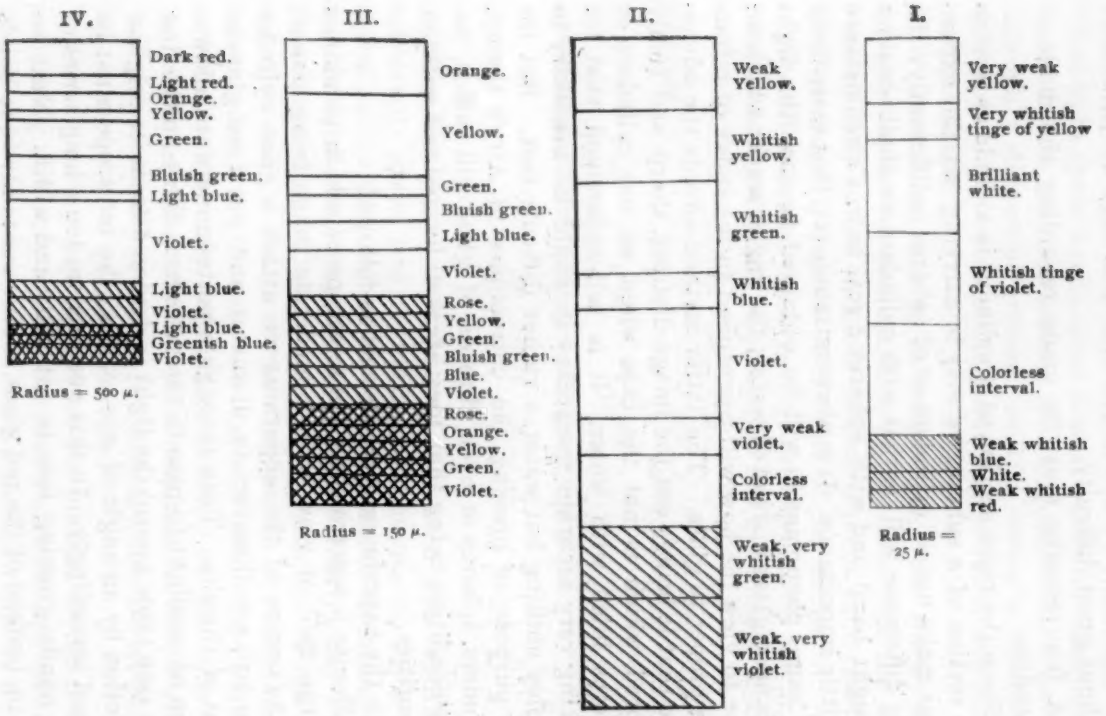


FIG. 3.—Four diagrams showing calculated rainbows (Pernter).

have to do with the phenomenon of diffraction: the further conclusion is easily drawn that in case we use white light, the different colors will overlap and thus give rise to mixed hues. Removing the glass and allowing the sunlight to fall upon the slit a phenomenon is observed which is essentially the same as the rainbow seen in nature. For we observe a long series of bands: the first shows red on the one end followed by yellow, green and violet (first bow), and the others (secondary bows) are a partial repetition of these hues.

We now proceed to the second part of the experiment. Instead of a cylinder measuring 2 to 3 millimeters in diameter, we use one of (say) 1 millimeter or a well-shaped glass-thread. The contrast is striking. The bands are broader, the colors less saturated and partly of a different succession; the blue, which before could hardly be noticed, is now prominent. Hence the phenomenon depends on the size of the cylinders. This conclusion, applied to the rainbow as it occurs in nature, gives a positive proof that the width of the entire bow, the width of the single colors and their succession, necessarily depend on the size of the raindrops. This we can understand better from *Figs. 4* and *5*. They show how the intensities of the different colors are superposed. *Fig. 5* especially gives us a fair idea of the final result—a white rainbow. (Cfr. No. I of *Fig. 3*.) In the first set of intensities the maximum of all the colors occurs almost in the same spot, hence white is produced in the main part of the principal rainbow. Between the first and second set all the colors have their minimum intensity (= 0) nearly at the same place, which explains the dark space between the principal and the first secondary bow as shown in No. I of *Fig. 3*.

Thus far the experiment affords a general proof for the correctness of Airy's theory; its performance requires neither a skillful experimentalist nor a well-furnished laboratory. However, a strictly scientific confirmation of the theory should be derived from experiments carried out with cylinders or drops of water, since it is water which in nature gives rise to the rainbow. W. H. Miller was the first to

make experiments in this line; afterwards they were taken up especially by Mr. Mascart, who in 1888 published his extensive and exact researches in the "Comptes Rendus." But since in the latter experiments only monochromatic light was made use of, Mr. Pernter saw himself necessitated to employ white light in order to have his calculations corroborated by experiment. Since their results demonstrate the accuracy of Airy's theory we think it proper to present them in their essential features.

CALCULATIONS FOR $a = 500\,\mu$			EXPERIMENTAL RESULTS FOR $a = 500\,\mu$		
ANGLE.		X	ANGLE		OBSERVED COLOR.
deg.	min.		deg.	min.	
42	10	red			
42	0	red			
41	50	red			
	40	yellow	41	36	Maximum of intensity.
	30	green	41	35	Limit between yellow and green.
	20	green			
	10	green	41	7	Limit between blue and violet.
	5	blue	41	5	Beginning of the first violet.
41	0	violet			
40	55	violet			
	50	violet	40	50	End of the first violet.*
	45	blue	40	45	Beginning of the second violet.
	40	violet			
	35	violet			
	30	rose	40	30	Middle of the second violet.
	25	rose	40	25	End of the second violet.
	20	blue			
	15	bluish green			
	10	blue			
	5	rose	40	2	Middle of the third violet.
40	0	rose	40	0	End of the third violet.
39	55	blue			
	50	bluish green			
	45	blue			
	40	rose			
	35	rose	39	38	Middle of the fourth violet.
	30	blue			
	25	blue			
	20	violet			
	15	rose			

* Average of three values.

A cylindrical water-jet was passed through the centre of the spectrometer and illuminated by sunlight admitted through a very narrow slit of the collimator. The angles of the different colors were accurately measured. Of course it was a difficult task to produce a jet exactly equal to that assumed in the calculations, hence Mr. Pernter sometimes had to content himself with an approximate value. There was also some difficulty in bringing the water-column to the exact center of the apparatus; this might have caused an

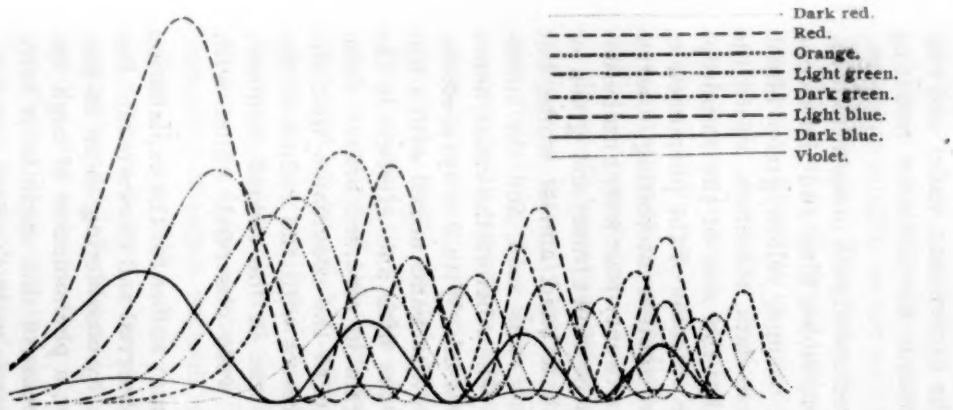


FIG. 4.—It shows how the bands of eight different colors overlap in case the radius of the drop measures 250 μ. (Pernter.)

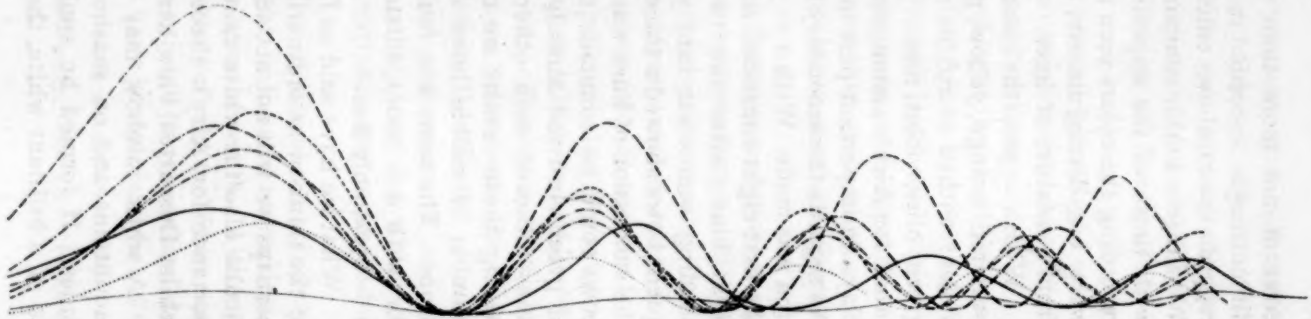


FIG. 5.—It shows how the bands of eight different colors overlap in case the radius of the drop measures 25 μ. (Pernter.)

error of not more than $\pm 4'$. Notwithstanding all these difficulties he succeded in getting quite satisfactory data as regards the rainbows calculated for $a = 500\,\mu$ and $a = 250\,\mu$. We subjoin a table comparing the results of the calculations with those of the experiment. It is to be noted that in recording the colors seen in the experiment, violet and rose were not distinguished; x stands for the color resulting from a mixture of hues.

With $a = 500\,\mu$ the colors succeeded each other as follows: pure red, orange, yellow, green, violet, blue and the second violet; with $a = 250\,\mu$: red, orange, yellow, green, bluish green, blue, violet, rose. These data, therefore, afford the proof for Airy's assumption that the size of the raindrops plays an important part in the outcome of the phenomenon. As regards the secondary bows another interesting observation was made. With $a = 500\mu$, twenty-four were numbered: the first eight consisted solely of violet (rose) and green or light blue; after the twelfth a band almost white, but shading somewhat into yellow, was seen, and the subsequent bows showed a trace of yellow before the violet, hence the succession of hues was reversed. With $a = 250\,\mu$, eleven bows could be counted: the white band mixed with a tint of green occurred already in the fifth and afterwards the hues followed each other again in reversed order. Comparing these results we come to the conclusion that the number of visible bows decreases with the radius of the drops. The same was found true in the second rainbow; for with $a = 500\,\mu$, thirteen were observed, whilst with $a = 250\mu$, only five.

What we have said so far, may suffice for the explanation of the rainbow as ordinarily observed and known to all. But perhaps we will not act amiss by considering more in particular also the white rainbow, a phenomenon of high importance for Airy's theory, because this explains it fully, whilst Descartes' view here leaves us in the dark.

A white rainbow may not infrequently be observed on mountains and on seashores, particularly in northern latitudes. If formed by sunlight, the principal bow is made up of a brilliant white, the margins are somewhat colored,

the outer showing a yellowish and orange-red, the inner a bluish-violet tinge. The secondary bows, if any occur, are separated from the principal arch by a dark space and their colors succeed one another in a reversed order; for a bluish seam lines the dark interval, whilst the red lies toward the inside.

Since a white rainbow may be formed by sunlight as well as by moonlight, we are now to investigate the causes of the phenomenon. They may be reduced to three.

The first is the weak intensity of light in case of the lunar rainbow. Since it is a known fact,* that by lowering the intensity of a color beyond a certain limit, each color as such ceases to make its own impression and appears as a grayish white, we easily understand that on account of the weak intensity of moonlight the lunar rainbow shows a whitish appearance.

It would be very interesting to find out under what conditions of intensity the rainbow-colors produce no effect on our visual organs. However, this question cannot be answered with certainty. Messrs. Abney and Festing, it is true, have shown experimentally when color disappears from our sight, but these data cannot be applied to the rainbow. First, there is no known standard to which we could refer the intensity of the rainbow-colors. Moreover, since daylight, which is variable in brightness, mixes with the colors, the sensation itself will thus be differently affected. Finally, because the air as regards dryness and moisture and the number of drops contained in a cubic meter (on which the intensity of colors depends) are variable, any estimate falls short of the reality.

A second cause is the inequality in size of the raindrops, whereby the different colors might be caused to overlap in such a way as to produce the sensation of white. Although we do not deny that this is possible, yet, since a white rainbow was never observed during a rainfall (it is assumed that it begins to rain when the drops have become at least 100μ

* Cfr. Abney and Festing, "Color Photometry," Part III; "Philosoph. Transact.," London, Vol. 183 (1892), p. 537.

in diameter) we need not dwell any longer on this explanation.

Very small drops are the third cause.

White occurs in every rainbow, whether in the principal or in a secondary one; however, we restrict the designation of "white rainbow" to those in which white is really prominent, without excluding differently colored shades.

Before we enter upon the explanation of this phenomenon, we must answer the question, in what case a mixture of colors gives rise to the sensation of white. Since every mixture of colors results into a pure spectral color and white, it is to be ascertained what percentage of each is required to make any color as such disappear. Mr. Abney proved that this percentage is the same for all colors, each disappearing in case a white light seventy-five times stronger be added. This means that white only then appears when the intensity of the color is 1·3 per cent. of that of white. At first sight this law seems to import the impossibility of a truly white rainbow since the calculated intensity of x is never as low as 1·3 per cent. The utmost we could expect is a sensible whitish shade in case x reaches 4 per cent. and below. Hence in our explanation we have to show that x reaches the limit stated by Mr. Abney.

We must bear in mind that Mr. Abney made his experiments in a dark room, whilst the rainbow is formed in full daylight and sunshine; whence it follows that since the sunlight is white and intense in comparison with the colors of the rainbow, the percentage of x is lessened whilst that of white is increased. If, therefore, the intensity of x be near the limit, 1·3 per cent., and if daylight be added, we understand that a truly white rainbow can be the outcome.

A series of interesting experiments, carried out by Mr. Pernter, confirm this explanation.

By means of ordinary atomizers he produced a fine water-spray (the size of the drops could not be measured on account of the small quantity of water), sunlight was allowed to fall upon it, and the phenomenon observed comported well with the raindrops figured out for very small drops. Red was wanting in the principal rainbow and the orange

showed a distinctly yellowish tint. Then very whitish shades of green and blue were seen, after which the violet came out more clearly as such. In full daylight these colors certainly would have given the rainbow represented in No. I of *Fig. 3;* this conclusion is corroborated by the large extension of the principal rainbow.

In another experiment the water-spray was dense enough to find the exact size of the drops. They measured 5·3 in radius. When sunlight fell upon the cloud of water-dust a splendid white rainbow was obtained, owing, without doubt, to the white sunlight which, although admitted in small quantity, increased the percentage of white enough to reach the limit for x. With $a = 8{\cdot}4\ \mu$ a white rainbow was yet obtained, the white band, however, had already decreased in width.

THE TECHNOLEXICON OF THE SOCIETY OF GERMAN ENGINEERS.

In the beginning of 1901, the Society of German Engineers (Verein Deutscher Ingenieure) began the compilation of a universal dictionary in the three languages, English, German and French. This undertaking has met with general approval and has received assistance from all quarters at home and abroad. Societies and individuals have responded generously to the invitation to collaborate and have proved their interest by the transmission of collections of technical words made by them or by promising such in the near future.

Up to now (May, 1903) there are 341 societies (42 in English, 272 in German, and 27 in French speaking countries) co-operating in the work, either by the systematical collection of technical expressions of the specialties represented by them or in other ways, especially by the acquisition of collaborators and by placing technical publications in more than one language at the disposal of the "Verein," as catalogues of firms, inventories, piecelists of machines, handbooks, etc. Through these societies the *Technolexicon* has found helpers in Great Britain, Germany, France, the United States, Austria, South Africa, India, Australia, Belgium, Canada, etc.

Among the English societies we name the following: The Institution of Mechanical Engineers, London; the Institution of Electrical Engineers, London; the Junior Institution of Engineers, London; the Society of Chemical Industry, London; the Institution of Mining Engineers, Newcastle on Tyne; the Iron and Steel Institute, London; the Society of Architects, London; the British Optical Association, London; the Optical Society, London; the Cycle Engineers' Institute, Birmingham; the South African Association for the Advancement of Science, Cape Town and Johannesburg; etc. Of American societies may be mentioned the American Society of Civil Engineers, New

York; the American Society of Mechanical Engineers, New York; American Railway Engineering and Maintenance-of-Way Association, Chicago; the American Chemical Society, Brooklyn; the Western Society of Engineers, Chicago, etc. In Germany there is first of all the large majority of the local sections of the Society of German Engineers; then there are Verein Deutscher Eisenbahnverwaltungen, the Verein Deutscher Chemiker, the Zentralverband der Preussischen Dampfkesselüberwachuugsvereine, the Verband Deutscher Patentanwälte, the Deutsche Schiffbautechnische Gesellschaft, and many others. In France we mention: Société des Ingénieurs Civils de France, Paris; Association Amicale de Anciens Élèves de l'Ecole Centrale, Paris; Société Internationale des Electriciens, Paris; Société d'Encouragement pour l'Industrie Nationale, Paris; Société Française de Photographie, Paris; Syndicat Général de l'Industrie des Cuirs et Peaux de la France, Paris, etc.

Assistance has so far been promised—and partly already rendered—by 2185 industrial establishments and individual collaborators.

The excerption of dictionaries like Sachs-Villatte, Muret-Sanders, Tolhausen, etc., and of thousands of firms' catalogues and price-lists, of handbooks, store-lists, piece-lists, customs tariffs, etc., has yielded 1,200,000 filled-out word-blanks up to May, 1903. To them will be added within the next two years the hundred thousands of words that the contribution of the collaborators will bring. For convenience sake the Society of German Engineers has provided handy note-books (each with three indices A—Z) for the collaborators to write their collections therein. These books will be called in by the editor-in-chief in the course of 1904. So far already 207 filled-out note-books have been forwarded voluntarily to the address given below.

As the contributions will be called in before 1904, all who wish to help in the compilation of the *Technolexicon* have still time and opportunity to assist in the preparation of their specialities. Contributions from all technical branches (including the handicrafts) are welcome, and it is obvious that small contributions from a host of various collaborators will be more useful than large ones compiled by a few men, who naturally cannot cover so many specialties. Attention is to be drawn to the fact that contributions in only one language are also most acceptable, though, of course, those in two or three languages are the most valuable, as also polyglot business-catalogues and other technical publications. Delayed contributions, if they arrive before the end of 1906, when we shall begin to print, can still be made use of.

The editor-in-chief will be pleased to give any information wanted. Address: Technolexicon, Dr. Hubert Jansen, Berlin (N.W. 7), Dorotheenstr., 49.

SOLDER FOR ALUMINUM.

A solder for aluminum, which forms the subject of a recent United States patent, consists of aluminum 5 parts, antimony 5 parts, and zinc 90 parts. To make it harder, use a little more antimony and a little less zinc. The solder is prepared as follows: The aluminum is first melted in a pot. The zinc is then added, and when this is melted the antimony is added. The metal is then thoroughly fluxed with sal-ammoniac. When the surface of the metal is quite clear and white it should be poured into sticks ready for use, the cinder being first removed.—*Scientific American Supplement.*

The Induction Motor and Its Engineering Capabilities.

[*A Thesis Presented to the University Faculty of Cornell University for the Degree of Doctor of Philosophy.*]

By George L. Hoxie.

(*Concluded from page 290.*)

Part Three.

Engineering Capabilities of the Induction Motor.

In General.—It is the intention of the writer to discuss very briefly in this section the applicability of the induction motor to a few classes of service which may be deemed typical. In some of these cases the induction motor is an almost ideal source of power, and in some others it may be made to do the work, but only after a considerable amount of trouble. The cases which will be considered are as follows:

(*a*) Driving of manufacturing establishments which demand a service similar to that demanded by a cotton mill.

(*b*) Driving of shops similar to machine shops.

(*c*) Operating pumps.

(*d*) Operating fans.

(*e*) Driving other electrical machinery or operating as frequency changers.

(*f*) Operating cranes or elevators.

(*g*) Operating electric railways.

It may be said in general that for any service requiring a nearly constant speed the induction motor will be found suitable. For a large starting torque, together with a good speed regulation, special devices may be added. For a service requiring a variable speed, and a torque varying inversely with the speed, the induction motor is not primarily well suited and it is only with difficulty that it can be adapted to such a service.

The electrical rivals of the induction motor are three, namely, the alternating current synchronous motor, the direct current shunt motor, and the direct current series motor. For many of the uses to which the shunt motor is put the induction motor is equally good and, in some cases, preferable. The latter has about as good a speed regulation as the former, and this may be obtained with a fair starting torque and a moderate demand for current. The induction motor has no parts demanding such constant attention as is required by the commutator of the direct current machine. The efficiency is about the same in each case. The advantages of current supply and distribution are with the polyphase currents required by the induction motor.

The alternating current synchronous motor has several advantages over the induction motor. Its speed regulation is absolute and its efficiency is somewhat better. The power factor of the synchronous machine may be made unity or it may be made to draw a lagging or a leading current, as is desired. On the contrary, such a machine is in general not self-starting and, if it loses at all in speed, say on a heavy overload, it will stop. In addition, the synchronous motor requires a direct exciting current.

The direct current series motor gives a torque approximately changing inversely with the speed. It is therefore well suited for traction work of all kinds, and for cranes, hoists, elevators, etc. The induction motor could not be a competitor at all for this class of service, except that its small cost for maintenance and the flexibility of the polyphase system which it uses, have led to the development of special devices by means of which a motor not primarily suited to a service may yet be made to answer its requirements satisfactorily.

(*a*) *For Driving Cotton Mills.*

Cotton machinery requires a drive that shall run at the most uniform possible speed. It is quite desirable that the motive power shall be self-starting, but frequent starts and stops are not necessary. It is very undesirable that an electrical machine should be used about which arcs or sparking

may occur. It is sometimes desirable to start up all the machinery of the establishment together, as is the general practice where the power is transmitted by shafting and belts.

All of these conditions point toward the employment of the induction motor, and such motors are largely used in practice. It has no serious competitor in this field, except perhaps the synchronous motor, and the difficulties of starting almost preclude the use of the latter.

(*b*) *For Driving Machine Shops.*

In a machine shop the conditions are quite different from those just discussed. There are usually tools of quite different characteristics to be driven, and many of the tools must be capable of operating at any one of several speeds. Two methods of electrical driving are in use and are found quite satisfactory. They are known as "group driving" and "individual driving." The former system is one that resembles to a considerable degree the method, formerly universal, of driving from a single source of power by means of shafting and belts. The tools are arranged in groups and each group is driven from a single motor. Usually a short-line shaft is used for each group and each machine is supplied with its own countershafting and belts. The number of tools driven from a single motor may be many or few, according to circumstances. Some groups may require the use of a large motor, while other groups may consist of single tools. The use of the group system, as a whole, does not therefore preclude individual driving in particular instances, though it usually involves an electrical installation that is not the best possible one for individual driving. Since electrical driving has for the most part been introduced into shops formerly driven through shafting, the method of group driving has merely involved a division of motive power into several units and a change to electric motors for those units. The expense of such a change is not great and the shock to the conservatism of owners and employees is a minimum. Group driving is therefore now in extensive use.

With group driving, the motor requirements are for a constant speed, some starting torque, and a minimum of attendance. These requirements are met in the best possible manner by induction motors, and, except in special cases, nothing else should be considered for a new installation.

What is known as individual driving means the employment of a separate motor for each tool that is to be driven. The advantages of this system are many and great. The initial expense is, however, considerable, and the old shafting, counter-shafting, belts, etc., that are usually in place, cannot be used. The conservatism of owners and managers has tended to prevent such a radical departure from the earlier practice, and for these reasons the system of individual driving has not been adopted to the fullest extent. In the shops of a few electric companies, however, elaborate installations of this sort have been made. In these cases the equipment serves as a valuable advertisement, and is useful from this point of view as well as for its intrinsic merits.

The requirements of a motor for individual driving differ with the character of the tools to be driven. In most cases not only is a wide range of speed necessary, but the tool must be capable of operating continuously at any of its possible speeds. The induction motor may be used in these cases, and the power transmitted through cone pulleys, back gears, etc., but it is not suited for the most direct connection to the work, and it cannot compete, for convenience in this respect, with the multiple voltage, shunt motor system. For the tools that run always at one speed, the induction motor is of course entirely satisfactory, but since only one system is likely to be installed in any given shop, and that one should be the one best adapted to all requirements, it is not likely to be found desirable to use this motor in machine shops where the electric drive is to be of the most complete character. It probably will be found best for those shops in which group driving, or driving from counter-shafting, is to be used. With certain wood-working machinery the induction motor is much to be preferred, since there is little

danger of its starting a fire, even when surrounded by combustibles.

(*c*) *Operating Pumps.*

For the electrical operation of pumps, the induction motor leaves little to be desired. It is particularly satisfactory when it is necessary to run the pump in an out-of-the-way place that is seldom visited. Large numbers of these pumps with induction motors are used for irrigation in the Western regions, and in many cases they are started and stopped from distant points, and visited only at infrequent intervals for the purpose of supplying oil. No other motor is in any degree so satisfactory for this as is the induction motor.

(*d*) *For Operating Fans.*

Much the same advantages are found for the induction motor when a fan is to be operated as were mentioned in the preceding paragraph. The ventilating system of a building must operate with as little attention as possible, and except in the case of the largest buildings, such attention as can be had is likely to be of a rather unskilled character. Between two motors rendering equally efficient service, one of which starts or stops with the throwing of a switch, and requires no other attention beyond an occasional oiling, while the other requires a little more trouble to start and stop, the same oiling, and constant attendance upon a commutator, there can be little question which will be chosen. The advantages of transmission besides are all on the side of the induction motor.

(*e*) *For Driving Other Electrical Machinery.*

Owing to the great advantages of polyphase alternating currents for the transmission of energy, few large generating stations are now built that do not involve a generation of such currents. At the same time it is practically necessary in many cases, that some form or forms of direct current shall be supplied, and occasionally it is desirable to make use of alternating currents of more than one frequency. The generating machinery, however, must be installed in

large units if the best economy is to be obtained, and besides this, the simplicity and ease of operation are much increased if all the generators are of the same type. These conditions lead in many cases to the installation of generating units to supply polyphase alternating currents only; the other forms of electrical energy being obtained from auxiliary machinery. This auxiliary machinery may be divided into three general classes: (1) Rotary Converters; (2) Motor Generators; (3) Frequency Changers.

Motor Generators.—Motor generators may be driven from any kind of electrical supply and may furnish any sort of electrical power. The only varieties that are of any present interest are those which are driven from a source of polyphase alternating currents. In this case there are two possibilities. They may be driven by synchronous motors, or they may be driven by induction motors. Each method has some advantages, and it is only a special study of each particular case that will enable one to decide which is best. The advantages of the induction motor are its ease of starting, and its small charge for attendance and repairs. Since the generator side of the motor generator set may always be brought up to speed before being loaded, it is unnecessary to use a motor with wound secondary, or slip rings.

The advantages of the synchronous motor are its absolute uniformity of speed (supposing the supply to have a constant frequency), and its high and regulable power factor. It has also a slight advantage in efficiency.

Considering first the question of starting, we find that this is not one of serious difficulty in either case. Since the starting is not done under load, it is possible to start up the synchronous motor by the induction motor action due to eddy currents induced in the pole pieces, or in the copper bridges sometimes used between the poles, or surrounding the poles. It is also possible to start the synchronous motor by means of a small auxiliary induction motor. In some cases, too, it is possible to start a motor generator set from the generator end. It is therefore true that, so far as the question of starting goes, there is no very serious objection to the synchronous machine.

As to speed regulation, the synchronous motor is preferable, but not greatly so, since an induction motor for this purpose need not have a high starting torque, and it may therefore be given an exceedingly good speed regulation. Furthermore, a slight drop in speed with increase in load may often be compensated for by compounding the generator end. This may always be done where the generator is a direct current machine for constant E.M.F. The direct current machine for constant current is largely run motor driven, but this type of machine does not demand an exceedingly close speed regulation.

The item of attendance and repairs is less for the induction motor, but the attendance feature is of small importance, since the generator end of the machine is always such as to require skilled attendance on its own account. The repairs should be quite small with either type of machine.

Concerning the power factor the advantages are greatly in favor of the synchronous motor. The power factor of this motor may be controlled by varying the field excitation. In some instances it has been found possible to keep the power factor on a line near unity, even when it was supplying current to induction motors, by drawing a leading current from it to synchronous motors. The value of this feature of the synchronous motor depends very largely upon the local conditions under which the plant may be installed. This is the only item of great importance that is in favor of the synchronous motor.

A minor feature which is to the disadvantage of the synchronous motor is its requirement of an auxiliary direct current exciter.

Frequency Changers.—The only practical type of frequency changer now in use is one of the induction-motor class. Such a frequency changer is in part an alternating-current transformer and in part an inverted induction generator.

Let us imagine that the secondary of an induction motor is blocked so that it cannot turn, and that the primary is excited normally by polyphase currents. We will assume that the motor has a wound secondary, the circuits of which terminate in slip rings. Under these circumstances the

motor will act like a simple alternating current transformer with its secondary circuit open. If a resistance load be connected to the slip rings, the motor will act like a transformer that is delivering current, but in this case the regulation of the transformer would not be good, since it would have considerable magnetic leakage. It might be so used, however, and would give fair results if of proper design. The primary and secondary frequency would of course be the same under these circumstances, while the ratio between the primary and the secondary E.M.F's would depend upon the number and arrangement of the turns. We may assume this ratio to be unity in a particular case.

If the motor is permitted to turn at half-speed we shall have an alternating current transformer, but one in which the secondary frequency is half that of the primary. The secondary E.M.F. is also reduced one-half. If the secondary be mechanically driven at half-speed in the opposite direction, its frequency and E.M.F. (neglecting leakage) are increased one-half.

Since the mechanical motion, when the secondary is driven backwards, is in a direction opposite to that of the resultant primary flux, it follows that electrical energy is transferred from the primary to the secondary. For the same reason it follows that the mechanical work done in driving the secondary also appears as electrical energy in the secondary circuit. If we neglect various losses, we may say that the part of the total energy that comes from the primary is, when compared with the total energy, in the ratio of the primary frequency to the secondary frequency. Also the ratio of the power supplied mechanically, to the total power output of the secondary, is the ratio of the frequency added by the mechanical driving to the total frequency of the secondary.

If, therefore, the primary frequency in a given case be 25 and the secondary frequency is to be 50, the secondary must be driven backward at what would be full synchronous speed in the opposite direction. The output of the frequency changer would be (neglecting losses) half supplied from the primary and half from the driving motor. The secondary

conductors would have twice the cross-section of those of the primary if the windings were otherwise alike.

Frequency changes are generally driven by synchronous motors. They are not particularly satisfactory commercially except in special cases, since the large magnetic leakage makes regulation difficult.

(*f*) *For Operating Cranes, etc.*

The induction motor would probably not be used upon cranes and similar classes of work, except for the great desirability of polyphase motors for other work and the inconvenience of maintaining two separate systems of electrical supply. These considerations have, however, led to the use of the induction-crane motor in many cases. The performance of machines of this sort, as to drop in speed under load and current and power required for light running, are given in the preceding table. The motors, if of squirrel-cage construction, are built with high resistance end rings, and while they give, therefore, a large torque they are not very efficient machines. In *Fig. 19* is given a comparison of the efficiency curves of a series, direct current-crane motor, and an induction motor, built for crane work and of about the same capacity as the former. The induction motor is one having a squirrel-cage rotor. The efficiency of the direct-current machine is considerably the better through its entire range, and that motor has a wide range of operation over which its efficiency is quite good. The induction motor reaches its maximum efficiency very abruptly. Both of these motors were made by the same company.

Speed variation in a motor of this sort is accomplished by the use of a special compensator that provides varying impressed pressures. This is not an efficient means of regulation, since the torque of such a motor varies with the square of the impressed pressure. Crane motors with wound secondaries may be made to give a variable speed by the variation of a resistance in series with the secondary. This again is inefficient, though it provides a highly satisfactory torque curve. The motor is not so desirable mechanically

since this method of control is considerably more complex. The motor with squirrel-cage secondary is probably the more desirable, on the whole, for this class of work.

It must not be supposed that the induction motor has no intrinsic advantages for crane work over the series direct

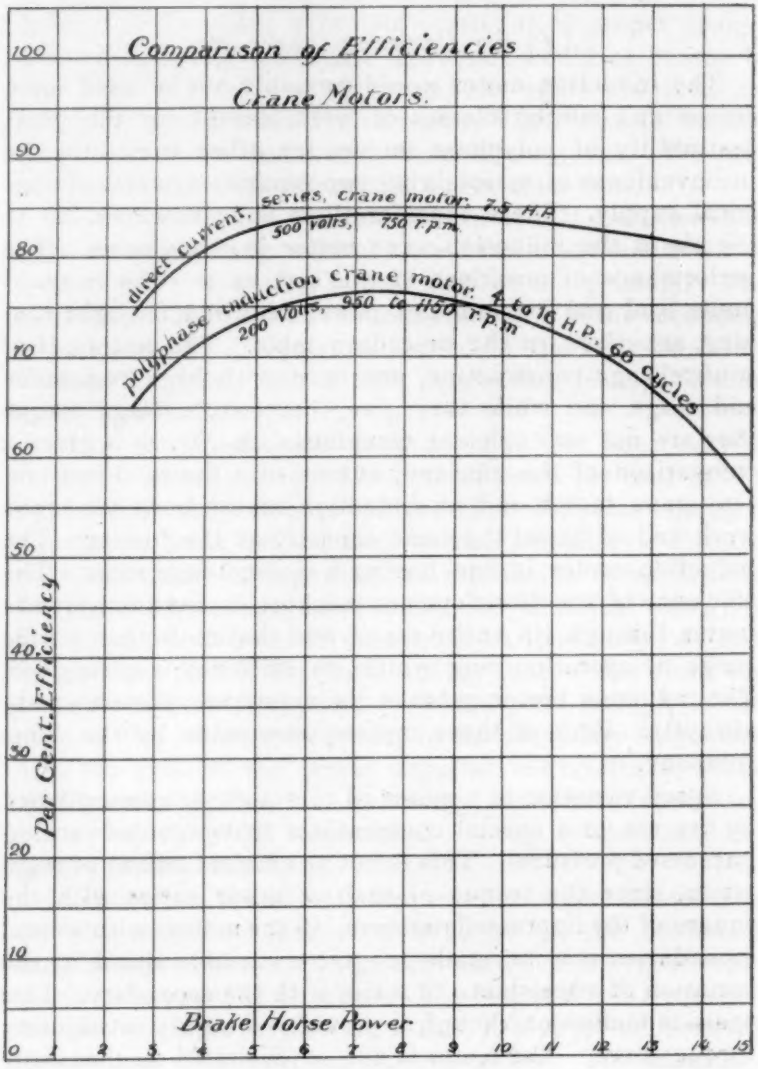


FIG. 19.

current motor. It has the very great advantages of simplicity and the capacity to withstand a great amount of ill treatment. Its performance otherwise is not so satisfactory as is that of the series motor.

(*g*) *For Operating Electric Railways.*

It is very hard to express an opinion as to the adaptability of the induction motor to general railway work. The actual use of this motor has been quite limited, and such installations as have been made are too recent to be accurately judged. No examples of this sort are found in the United States, and in Europe the roads which are so operated do not present problems at all resembling those which are most commonly met with in this country. The promoters of the induction railway motor, or of a polyphase system for electric railway operation, have evidently a considerable degree of confidence in the system. One large European firm (Ganz & Co.) recently made a vigorous effort to obtain the contract for the electrification of two underground railway systems in London, and their proposition covered a complete system for the polyphase operation of those roads, induction motors being used exclusively. It seems probable that such a proposal would not have been seriously made by a large and responsible company had any doubt as to its feasibility existed in the minds of their engineers.

It is easy to state the theoretical advantages of the induction motor and of the direct current motor. When one turns to the practical aspect of the question, however, one finds statements from eminent experts that are flatly contradictory as to matters of fact. The final answer to these disputed questions can only come after the completion and operation (or failure) of a road resembling that of a surface or underground line in one of our great cities.

From the theoretical point of view, we may study the subject under the following heads:

(1) Initial Cost.
(2) Methods of Control.
(3) Maintenance of Schedule.
(4) Operating Convenience.

(5) Efficiency.

Many other things might be discussed, but these are sufficient to bring out a comparison of the theoretical qualities of the two classes of motors.

(1) *Initial Cost.*

Both systems are likely, in large installations, to make use of the same type of generating stations. The main transmission lines would probably also be the same. With the direct-current system expensive sub-stations are necessary and the lines from the sub-stations to the motors must have a large current-carrying capacity.

With the polyphase system any sub-stations used are pressure-transforming stations only and are comparatively inexpensive. They do not require attendance. The lines from the sub-stations to the motors may be much smaller than for direct currents, since the cars may safely take current at high pressure.

The induction motors should be the cheaper, although it is not practicable to use motors having squirrel-cage secondaries. The controlling devices are very complex in either case. Those for use with direct currents have been standardized and are manufactured in large quantities. If polyphase motors were to become widely used for traction purposes it seems probable that their controllers could be manufactured more cheaply than those for direct current motors. From the standpoint of initial cost, therefore, the advantage should lie with the polyphase system. This conclusion is verified by a comparison of the bids for the electrification of the two London roads previously mentioned.

(2) *Methods of Control.*

A traction motor must be capable of giving a powerful torque on starting or at low speeds. To secure this with a single motor, if of the induction type, it is necessary to insert a large resistance in its secondary. The motor is then quite inefficient. This method may be compared with the resistance control of a direct-current series motor.

A system known as the "cascade system," or as a system

of "concatenated control," has been somewhat used and is strongly recommended by its sponsors. This system is somewhat analogous to the series parallel control which is so generally used with direct-current motors. In the cascade system two motors are used, and one of them is supplied with current from the secondary of the other. Additional regulation is secured by a resistance in the secondary of the last motor. This arrangement is only intended to be used between zero and half-speed of the car. At half-speed of the main motor its secondary frequency is half that of the primary, and the second motor, if similar and running at the same speed, is running at synchronous speed. The first motor is really acting partly as a motor and partly as a frequency changer for the second motor. A resistance in the secondary of the second motor produces about the same effect as if it were placed in the secondary of the first motor, so far as the operation of that machine goes. This is more nearly true as the magnetic leakage of the two motors is diminished. It is, therefore, the practice with the cascade system to use motors having as few poles as possible and an excessively short air gap. There is some question as to the safety of this short air gap for traction work. The frequencies used are low, often 15 cycles per second.

Since the induction motor, when driven slightly beyond synchronism, becomes a generator, it is possible to return power to the line on long down grades by permitting the car to run downward a few per cent. faster than upward. This property seems to be of value only in special cases.

(3) *Maintenance of Schedule.*

Since for ordinary running the motors must operate at a speed that is close to synchronism, trains operated by them will either run at a uniform speed or will stop. It therefore follows that if the motors are adequate to the work a train cannot lose time while running. This has been urged as a great advantage for the induction motor. It is equally true, however, that if a train be delayed from any cause, no time can be made up while running. Experience alone will determine which of these features is the more important.

(4) *Operating Convenience.*

The polyphase system possesses a great advantage, in that no attendance, beyond an occasional inspection, is demanded by its sub-stations. The motors have a further advantage in not possessing commutators. The controlling devices are essentially no more complex than those for direct currents. The necessary use of two sliding contacts, or trolleys, is a considerable disadvantage.

(5) *Efficiency.*

In considering the question of efficiency, the entire system should be considered in each case. The progress of the power from the generating station to the motors may be taken as the same up to the sub-stations. The power factors up to that point may be nearly alike if the rotary converters, which we will assume for the direct current system, be compounded. At the sub-stations, the direct-current system loses from 5 to 10 per cent. in the rotary converters. There is no corresponding loss in the other system. The loss between the sub-stations and the motors would rarely be less than 10 per cent. with the direct current, and would often be more. For the polyphase system this loss should be under 10 per cent. At the motors, the running efficiency should be about the same in the two cases. The starting conditions would favor the direct current.

In conclusion, it may be said that the use of the induction motor is undoubtedly increasing, and it is possible that it may ultimately be used for traction purposes upon the large trunk lines of the country.

ELECTRICAL SECTION.

Stated Meeting, held Thursday, April 23, 1903.

Some Practical Points in the Design of Direct-Current Dynamo-Electric Machines.

By Cecil P. Poole, Member A.I.E.E.

The design of direct-current dynamo-electric machines has become a somewhat hackneyed subject of late, and but for the fact that some of the practical points which it is the object of this paper to present have not hitherto been made public, the author would certainly not have the temerity to discuss the question.

It is, of course, well known that the output of a direct-current machine is limited in practice by either heating or commutation, or both. It has been the author's observation that commutation sets the limit in a larger proportion of machines than does heating, especially when the modern well-ventilated construction is used. That bad commutation is due to a combination of armature reaction on the field and the inductance of the coil undergoing commutation, is so well understood as to require no discussion. It is also a familiar fact that the interposition of resistance between the terminals of a coil short-circuited for commutation, as by using carbon brushes, or high-resistance commutator risers, etc., usually assists commutation by reducing the time constant of the short-circuited coil. But it is also true that machines of commercially practical character may be calculated beforehand with reasonable certainty as to their sparking limit, without taking into consideration the resistance of the coil connections and brush faces beyond using certain constants for machines which are to operate with carbon brushes, and others in the calculation of machines to be operated with metal brushes.

Since sparking is due chiefly to the counter E.M.F. of inductance in the coil undergoing commutation, or more properly the reactance E.M.F. of the coil, good commutation

depends primarily upon the strength of the field in the commutating zone near the tip of a field magnet pole. The ratio

$$\frac{e_c}{e_x}$$

is the very best criterion of sparkless commutation, e_c representing the average E.M.F. induced in the short-circuited coil during commutation, and e_x its average reactance E.M.F. during that period. The larger this ratio, the better the commutation. The value of e_c depends of course upon the strength of the exciting force that is applied to the air-gap, and negatively upon the weakening effect of the armature ampere-turns.

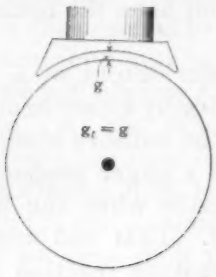


FIG. 1.

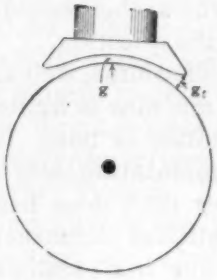


FIG. 2.

If there were no armature reaction, the flux density under a pole-tip might be considered approximately equal to the average density over the whole tip for pole-faces concentric to the armature core throughout their entire span, as in *Fig. 1*. And, representing the density of the commutating flux by B_c, the average density over the whole pole-face by B_p, the length of the air-gap under a tip by g_t, the principal air-gap under a concentric pole-face with slanted tip faces, as in *Fig. 2*, and the mean air-gap under a pole-face eccentric to the armature, as in *Fig. 3*, by g, it would be fair, for purposes of comparison, to assume the general relation

$$B_c = B_p \frac{g}{g_t}.$$

This might, of course, be reduced to $B_c = B_p$ for *Fig. 1*, but there is no gain in the reduction, in view of subsequent

developments. On this basis, the E.M.F. induced in a coil, and available for reversing the current in it against its self-induction, might be taken as

$$e_c = B_p \frac{g}{g_t} D \pi L_p 2t \frac{r.p.m.}{60} 10^{-8}$$

$$= \frac{\pi}{30} B_p \frac{g}{g_t} D L_p t \, r.p.m. \, 10^{-8};$$

in which

D = Armature core diameter.
L_p = Pole-face length, axially.
t = Turns per armature coil.
$r.p.m.$ = Revolutions per minute.

With a load, however, there will be $\frac{i w \lambda}{P}$ ampere-turns on the armature setting up a magnetic stress along the path indicated by the dotted line in *Fig. 4*.

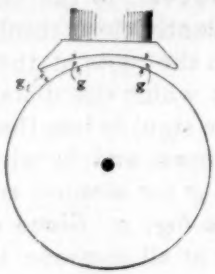


FIG. 3.

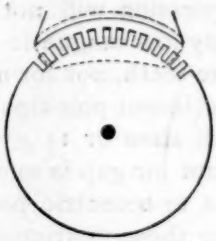


FIG. 4.

i = Amperes per armature wire.
w = Total number of armature wires.
P = Number of magnet poles.
λ = Polar arc ÷ pole pitch; in the case of skewed or pointed or rounded pole-tip edges, the *average* polar arc ÷ pole pitch.

These ampere-turns on the armature may be considered capable of producing a flux between the pole-tips and the armature core of a density,

$$B_a = \frac{i \, w \, \lambda}{K_p \, P \, g_t}$$

K_p being a constant the value of which depends on the degree to which the pole-piece and armature core are magnetized by the field winding, and the shape of the pole-tip edges. Ordinarily it ranges from unity to 1·05, and in most cases is negligible.

The net strength of the field under a commutating pole-tip, therefore, may be approximately expressed by the following equation:

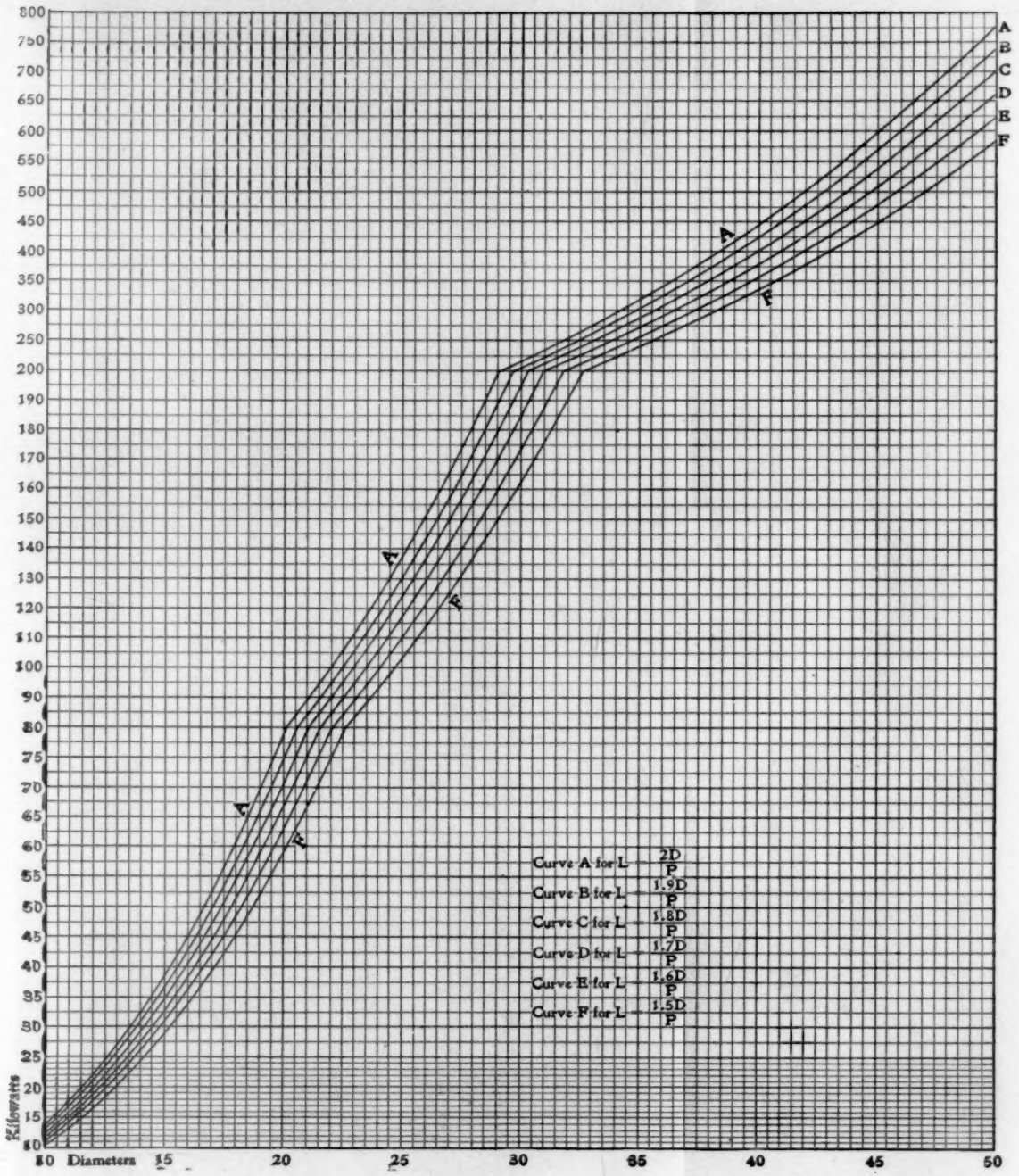
$$B_c = \left(B_p\, g - \frac{i\, w\, \lambda}{K_p\, P}\right) \frac{I}{g_t} \qquad (1)$$

Accordingly, the commutating E.M.F. induced in a coil undergoing commutation, may be approximately calculated by means of the formula:

$$e_c = \frac{\pi}{30}\left(B_p\, g - \frac{i\, w\, \lambda}{K_p\, P}\right) \frac{D\, L_p\, t\, r.p.m.\ 10^{-8}}{g_t} \qquad (2)$$

It is important to call attention, however, to the fact that this expression will not give sufficiently close results with relatively low magnetic densities in the roots of the armature core teeth, nor for machines in which the distance between adjacent pole-tips (of opposite sign) is less than $12\, g_t$ for small sizes or $15\, g_t$ for larger ones, and in which the maximum air-gap is more than $1{\cdot}5\, g$ for slanted edges as in *Fig. 2*, or eccentric pole-faces like *Fig. 3*. Since designs violating these restrictions are not at all common, the usefulness of the equations is not particularly impaired by having to observe them. It will be found good practice to make ratio of pole-tip proximity to pole-tip air-gap not less than 15 for machines having armatures 24 inches or more in diameter, and to reduce the minimum ratio progressively to 12 at a diameter of 18 inches, thus:

Armature Diameter.	Inter-polar distance ÷ Air-gap under pole-tip.
18 ins. or less	12
19 ins.	12½
20 ins.	13
21 ins.	13½
22 ins.	14
23 ins.	14½
24 ins. and over	15


Kilowatts
Diameters
A
B
C
D
E
F
Curve A for L = 2D/P
Curve B for L = 1.9D/P
Curve C for L = 1.8D/P
Curve D for L = 1.7D/P
Curve E for L = 1.6D/P
Curve F for L = 1.5D/P

Having a guide as to the magnitude of the commutating E.M.F., e_c, the approximate value of the ratio,

$$\frac{e_c}{e_x},$$

may be derived by following the method established by Messrs. H. F. Parshall and H. M. Hobart for estimating the reactance E.M.F., e_x. No method for determining the actual reactance of a coil under commutation has ever been made public, nor has any investigation of the exact manner in which the short-circuit current varies under different conditions ever been published, so far as the author's knowledge goes. Assuming a variation according to the sine law, which is admittedly incorrect, but is the only basis thus far suggested, the reactance E.M.F. of a coil undergoing commutation may be estimated by means of the formula

$$e_x = 6{\cdot}66\ L_a\ i\ t^2\ c\ \frac{D_c}{b}\ r.p.m.\ 10^{-8} \qquad (3)$$

in which

L_a = Gross armature core length.
i = Current per coil.
t = Turns per coil.
c = Maximum number of commutator segments simultaneously in contact with one brush.
D_c = Diameter of commutator barrel.
b = Thickness of each brush, if set radially.

The brush "thickness" is the chord of the arc of contact at the brush face, and is considered because it is more convenient to ascertain it than the length of the arc of contact, which would be the correct measurement if it were possible to deduce exact values for the other factors.

Substituting the equivalent for e_c given by equation (2) and that for e_x given by equation (3), and substituting

$$\frac{w}{2\,C}$$

for t (C = number of armature coils), after cancelling common factors, the following is obtained:

$$\frac{e_c}{e_x} = \left(\frac{B_p\,g}{i\ w} - \frac{\lambda}{K_p\,P}\right)\frac{D\,b\,C}{D_c\,c\,g_t}\cdot\frac{1}{31{\cdot}8} \qquad (4)$$

For use as a criterion of commutation, it is more convenient to write:

$$K_c = \left(\frac{B_p g}{i w} - \frac{\lambda}{K_p P}\right)\frac{D b C}{D_c c g_t} \qquad (5)$$

In machines having carbon brushes, the value of K_c should not be much less than the value that the derivation of the formula gives if the brushes are to have a slight lead; 30 is a good value for K_c in such cases, and with a considerable lead, the value of K_c may be as low as 20 for fixed brushes. For generators requiring adjustment of brushes for load variation (which is scarcely a commercial condition), the formula is not a reliable criterion owing to the wide differences in the values of B_d and B_c in different machines having the same values of B_p and g.

The author has applied this criterion to eighteen machines, and found it to be on the safe side by a moderate margin in fifteen out of the eighteen cases, and by a large margin in two of them; in the remaining case the formula failed utterly, and further investigation revealed the fact that the machine—a 125-volt, 1500-kilowatt dynamo—"ran sparklessly with fixed brushes *almost up to full load*," but the brushes had to be readjusted beyond that point. The exact proportion of full load carried without brush readjustment was said to be "about 80 per cent."

The formula indicates that the oft-repeated statement to the effect that the number of commutator segments should be as high as may be commercially practicable is not unconditionally correct. As an inverse indication of the number of turns per coil, the number of coils should, of course, be large, but after one has got down to one turn per coil, increasing the number of coils beyond a certain point is actually detrimental, as an analysis of the relations

$$\frac{b C}{D_c c}$$

and

$$\frac{B_p}{i w}$$

in the formula indicates. With a given brush thickness and commutator diameter, increasing C beyond the point

where it will change the value of c is disadvantageous, unless the machine is already well within the sparkless limit, in which case nothing will be gained ordinarily by increasing C. There is a minimum value for C, however, below which it is not safe to go on account of the amplitude of oscillatory reaction by the successively short-circuited armature coils upon the increased magnetic field in the commutating zone. Just what this limit is, the author has never found it necessary to determine. Mr. Kapp, the well-known European engineer, determines it by means of the formula

$$\frac{C\,g}{D\,(1+k)} \geqq 0{\cdot}6$$

for carbon brushes; k representing $b \div$ thickness of one commutator segment and mica strip. This, however, is based on certain conditions and criteria, and is not universally applicable. Nevertheless, it indicates a good condition if the machine is at all conventional in point of air-gap density, armature ampere-turns per pole, etc. Sparkless machines have been built, however, in which Mr. Kapp's constant was as low as 0·4. The author has found it usually sufficient to make the number of coils and commutator segments of a dynamo or a very large motor satisfy the criterion

$$C \geqq 1{\cdot}6\,P\sqrt[3]{E^2} \qquad (6)$$

E being the E.M.F. measured at the brushes, under full load. For small motors the number of segments may be much lower than this formula would make it, but small motors constitute an exception to almost every rule. For motors of moderate size, the number of segments may be

$$C \geqq 0{\cdot}8\,P\sqrt[3]{E^2} \qquad (6a)$$

and for motors of 30 to 100 horse-power

$$C \geqq 1{\cdot}2\,P\sqrt[3]{E^2} \qquad (6b)$$

All of these formulas are merely for general guidance in preliminary calculations to avoid starting out with too low

a value for C. If the machine, when finally completed on paper, meets the requirements of equation (5), there will be no probability of having too small a number of commutator segments.

Two short formulæ of criterion used by the author in "roughing out" designs are

$$\frac{B_p P g}{i w \lambda} \geqq 2{\cdot}5 \qquad (7)$$

and

$$\frac{B_p D b}{i c t D_c} \geqq 190 \qquad (8)$$

These values are not general, but one can easily deduce the proper values for any class of design, and they will serve admirably for preliminary criteria of other designs of the same class but different sizes.

The foregoing discussion is not intended, so far as formulæ (1), (2), (4), (5) and (8) are concerned, to apply to motors of the reversible class, such as street railway and elevator motors. Since in this type of machine the brushes are set midway between the point at which they would be set for rotation continuously in one direction and the position they would occupy for rotation continuously in the opposite direction, commutation cannot be assisted by means of the magnetic fringe under the "trailing" pole-tips. In motors of this class, successful commutation depends upon the smallness of the inductance of the commutated coil and the absence of interference by the field flux. Formulæ (3) and (7), therefore, are useful in designing reversing motors; the value of e_x in general should not exceed 3 and the value of the right-hand member in formula (7) should be 3·5 instead of 2·5. The author has found in actual practice that a minimum ratio of 4 invariably gives satisfactory results, even when the value of e_x is appreciably higher than 3, and rather than be restricted to this low figure, has adopted the expedient of using 4 as the minimum value of the ratio

$$\frac{B_p P g}{i w \lambda}.$$

DESIGNING AN ENTIRE STANDARD LIST.

In working out designs for special machines and most direct-connected machines, each case must be considered alone, as a rule. With a full list of belted machines to be worked out, however, much time may be saved by laying down at the outset certain general relations to be applied to the whole list, and this method also produces more consistent results than that of designing each machine separately. Nearly ten years ago,* the author advocated this practice, and also the separation of the formulas for output into two groups; one containing factors constant for the whole line, and the other including the variable factors. This method, however, is applicable only to a limited range of sizes, since it involves working with a constant pole-face density and peripheral speed. A modification of the older method that is applicable to a line of almost unlimited range has been devised by the author, and applied satisfactorily in several instances. This consists in working with the same magnetic densities in the iron paths of all sizes of machines and expressing as many values as possible in terms of the armature diameter.

The formula for electromotive force induced in any direct-current armature may be written

$$E = D\,\pi\,L_p\,B_p\,\lambda\,\frac{w}{p}\,\frac{r.p.m.}{60}\,10^{-8},$$

in which all symbols previously used have the same significance as before; p represents the number of paths through the armature winding, and the use of the reciprocal of this number as a factor in the equation makes it applicable to all commercial classes of armature winding. Inasmuch as the total current delivered by the armature is equal to $i\,p$, the formula for total generated power, in watts, may be written

$$W = D\,\pi\,L_p\,B_p\,\lambda\,i\,w\,\frac{r.p.m.}{60}\,10^{-8}.$$

Now it is entirely rational and practical to make the pole-face density and the speed proportional each to some power

Vide Electrical Engineer, December 6, 1893.

of the armature diameter. It is an obvious fact that high peripheral velocity is economical, but it has been demonstrated by experience that small machines cannot be given as great peripheral velocities as large ones; if a 10-kilowatt machine, for example, were operated at the same peripheral velocity as a 250-kilowatt machine, the speed rate in revolutions would be undesirably high, or else the diameters of the machine would be absurdly great. The author has found the relation

$$r.\,p.\,m. = 5{,}000 \div D^{\frac{2}{3}} \qquad (9)$$

to be practical over a wide range, and it has the advantage of entailing only simple arithmetical computation. The value of

$$D^{\frac{2}{3}}$$

may be taken directly from a table of squares and cubes, and the reciprocal of double this value, obtainable from a table, will be the revolutions per minute ÷ 10,000.

The pole-face density may be approximated in preliminary calculations by means of the equation

$$B_p = 34{,}380\, D^{\frac{1}{3}} \qquad 0)$$

Any coefficient between 33,000 and 36,000 will give good practical values; the value 34,380, which will seem academic at first glance, has been chosen for a purpose that will appear presently. Table I shows the results obtained from the application of these two formulæ through a pretty wide range of diameters, and will serve, in the author's opinion, to indicate their practical character. It is scarcely necessary to point out that the values thus obtained are not intended to be used rigidly. Whenever it is practicable to do so, the author adopts the nearest rational speed rate, as given in the table, but it frequently develops in the final refinements and compromises between conflicting *desiderata* that a slight departure from the preliminary speed value is advisable. The preliminary value for pole-face density nearly always must be modified somewhat to suit other conditions.

Substituting in the formulæ for E.M.F. and power the equivalents of *r.p.m.* and B_p as given by formulæ (9) and (10), one obtains the very simple preliminary equations:

$$E = 0{\cdot}09\sqrt{D}\,L_p\,\lambda\,\frac{w}{p} \qquad (11)$$

and

$$W = 0{\cdot}09\sqrt{D}\,L_p\,\lambda\,i\,w \qquad (12)$$

from which it will be evident that the coefficient in equation (10) was given its particular value in order to simplify the coefficient in these last two equations.

In order to get at the approximate armature dimensions without involving the likelihood of selecting dimensions that must be afterward changed radically on account of commutation considerations, equations (7) and (12) may be combined, substituting in the latter the second number of the following transposition of equation (7)

$$i\,w\,\lambda = \frac{B_p\,P\,g}{2{\cdot}532},$$

and expressing B_p in terms of D, as before. The value of g may be taken as

$$g = 0{\cdot}018\,D^{0{\cdot}8333} \qquad (13)$$

and using this equivalent, the following preliminary formula is obtained:

$$Kw. = 0{\cdot}022\,D^{1{\cdot}5}\,L_p\,P \qquad (14)$$

which will serve as a guide to suitable dimensions within the sparkless range. If a fixed ratio of $D : L_p$ could be adopted practically, the formula might be still further simplified, but this is not feasible. It is entirely practical, however, to adopt fixed limits between which $D : L_p$ may vary, thus:

$$L_p \geqq 1{\cdot}5\,\frac{D}{P};$$

$$L_p \leqq 2\,\frac{D}{P}.$$

The accompanying chart has been plotted for the ratios of $D : L_p$ between these limits, advancing in steps of 0·1, and is practically self-explanatory.

The actual diameter adopted will depend largely upon the current density in the armature conductors, the "space factor" of the conductors, and the magnetic density at the roots of the teeth. For belted machines it will suffice to assume tentatively a current density of 2,000 amperes per square inch of conductor cross-section and an apparent flux density of 150,000 to 160,000 maxwells per square inch at the narrowest part of the teeth, making no allowance for fringing. The ratio

$$\frac{\textit{Slot depth}}{\textit{Slot width}} = \frac{s}{h}$$

should not exceed 5 in very small machines, coming down to 4 in armatures of about 18 inches diameter. For larger diameters this ratio may be used also, but it is preferable to reduce it slightly whenever other considerations are favorable. Observing this restriction, the dimensions of a slot may be determined tentatively by allowing 180 mils in depth for insulation and $120 + 10\,n$ mils in width, in addition to the cotton insulation on the conductors, n being the number of conductors side by side in one slot.

Let B_t = Apparent density at teeth roots,
l = Net axial iron length of armature core,
j = Width of one core tooth at the root,
S = Number of slots in the core;

then the aggregate of the tooth widths all around the armature will be

$$S\,j = \frac{\pi\,(D + 2g)\,L_p\,B_p}{l\,B_t}. \qquad (15)$$

It is entirely practical to ignore the difference between the circumference of the pole-face circle and that of the armature core, and to assume the net iron measurement to be $0{\cdot}8\,L_p$. On this basis, and the assumptions previously made as to pole-face and maximum tooth densities, the following relation is derived:

$$S\,j = \pi\,K_t\,D^{1{\cdot}1667}, \qquad (15{\cdot}a)$$

the value of K_t being $43{,}000 \div B_t$. The value $0{\cdot}28$ will serve for all ordinary cases.

If one prefers to exercise some discrimination in the matter of the apparent density at the teeth roots and the relation between the net iron length of the armature core l and the axial width L_p of the pole-face, and *not* to ignore the difference between the circumference of the pole-face circle and that of the armature core, the relations between the various factors may be very conveniently tabulated by reducing each pair to a ratio. Thus Tables II and II-a give various values for

$$\frac{l}{L_p}, \quad \frac{B_t}{B_p} \quad \text{and} \quad \frac{Sj}{D + 2g}$$

within practical limits, the relations between which are identical with those determined by formula 15, π having been eliminated in calculating the values in the tables. These tables, therefore, make it practicable to determine the proper value for any one of the three ratios after having decided upon the value of the other two. Mechanical considerations usually dictate the value of

$$\frac{l}{L_p}$$

and also govern to a considerable extent the value of

$$\frac{Sj}{D + 2g},$$

and the application of the trial-and-error method is necessary only to a limited extent in order to obtain satisfactory relations between the three ratios.

Having determined the slot dimensions, the diameter of the core under the slots ($D - 2s$) will be known, and one has at once the formula

$$S = \frac{\pi (D - 2s - 0{\cdot}28 D^{1{\cdot}1667})}{h} \qquad (16)$$

for the available number of core slots.

When the size of the machine is such that wire of rectangular cross-section may be used, an appropriate depth of slot may be determined by means of the formula

$$s = 1 + 0{\cdot}125 \sqrt{D} \qquad (17)$$

and the slot width made to accommodate the thickness of the armature conductor, within the restriction previously set as to the ratio $s : h$. Allowing 90 mils for taping or other insulation on the outside of each group of conductors, the depth of each conductor expressed in mils will be

$$\frac{820 + 125\sqrt{D}}{2} - \text{insulation},$$

the "insulation" being the total thickness of the cotton insulation on the wire as it is received from the manufacturer. Taking this at 20 mils (10 mils on each side), the depth of a bare conductor, in mils, will be

$$u = 390 + 62{\cdot}5\sqrt{D} \qquad (18)$$

and at 2,000 amperes density, the bare thickness of the conductor, expressed in mils, will be

$$q = \frac{i}{{\cdot}78 + {\cdot}125\sqrt{D}} \qquad (19)$$

from which the width of slot may be derived. Allowing $120 + 30n$ mils for insulation,

$$h = n\left(\frac{i}{780 + 125\sqrt{D}} + {\cdot}03\right) + {\cdot}12 \qquad (20)$$

By making up tables or charts of values for s, h, q and u, corresponding to different practical values of D and i, one may choose immediately provisional slot and conductor-dimensions that will require very little if any modification in the final rounding off of each design.

At this point, the criterion for minimum number of commutator segments should be applied, and the ability of the armature to generate the required E.M.F. tested by the application of formula (6) and equation (11), obtaining temporarily a value for λ by using 2·532 as the constant in formula (7) and solving for λ. If the required minimum number of commutator segments cannot be obtained, or is only obtainable by exceeding it absurdly, or if the E.M.F. falls short, the proper diameter may be determined immediately, of course, by the ordinary "rule of three" method, thus:

$$D = \frac{C D_1}{C_1};$$

and

$$D = \frac{E^{\frac{2}{3}} D_1}{E^{\frac{2}{3}}_1};$$

the letters with subscript *1* indicating the values first obtained and the others the required values. Having the diameter, the new length may be readily determined by means either of the chart or of formula (14).

The thickness of a radial carbon brush may be assumed tentatively as

$$b = 3 g_t \frac{D_c}{D} \qquad (21$$

The diameter of the commutator cannot well be expressed in terms of D, since the current and the number of turns per coil must be considered in deducing the mean width of a segment, and the diameter should be kept as small as the size and number of segments will allow. With conductors arranged "two deep" in a slot, the number of segments will be

$$\frac{S n}{t} = C,$$

and the value of S cannot always be taken as exactly the number obtained by equation (16) because it must be appropriate to the class of winding used; moreover, the value of n will vary roughly as the E.M.F. of the machine, for a given diameter and output. But a preliminary commutator diameter may be deduced readily enough from the other data that will have been derived at this stage of the procedure, and formula (8) may then be applied, substituting the equivalents of B_p and b given by equations (10) and (21), thus:

$$\frac{g_t D^{0.1667}}{i c t} \geqq 0.00184.$$

If the result is not below the limit set by this formula, the more elaborate criterion afforded by formula (5) may be

applied, and the actual proportion of polar embrace, λ, deduced simultaneously, by substituting the values thus far determined and transposing the formula to solve for λ.

The radial depth of the armature core between the slots and the center hole may be determined by means of the equation

$$d = \frac{\lambda D^{1\frac{1}{2}}}{P} \qquad (22)$$

which has been derived by assuming a magnetic density of slightly less than 68,000 maxwells per square inch, a net iron length, axially, of 0·8 of the axial pole-face length, and the relation between pole-face density and armature diameter that has been carried through all of the calculations.

The cross-sectional area of each magnet core may be ascertained by means of one of the following equations:

For wrought iron,

$$M_w = \frac{1{\cdot}08\ \lambda\ L_p\ D^{1\frac{1}{2}}\ v}{P} \qquad (23)$$

For cast-steel,

$$M_s = \frac{1{\cdot}12\ \lambda\ L_p\ D^{1\frac{1}{2}}\ v}{P} \qquad (24)$$

For cast-iron,

$$M_i = \frac{2{\cdot}5\ \lambda\ L_p\ D^{1\frac{1}{2}}\ v}{P} \qquad (25)$$

in which v is the leakage or dispersion factor. These equations are based on densities of 100,000 maxwells per square inch in wrought iron, 96,000 in steel, and 45,000 in cast-iron. If one wishes to work with other densities, the proper coefficients may be derived thus:

$$\frac{108{,}000}{B_m} = K_m.$$

The leakage factor must, of course, be ascertained from previous experience, or estimated by comparison with some existing machine of similar dimensions and type. It is an utter waste of time to attempt to calculate it.

The length of a shunt field magnet coil parallel with the pole may be approximated by means of the formula

$$L_c = \sqrt{D} \tag{26}$$

from which the length of pole is readily obtainable for simple shunt-wound machines. The series coil of a compound-wound machine need not exceed one-half of the length of the shunt coil, except in the case of heavily over-compounded machines, such as railway generators. A very good guide as to total coil space parallel with the magnet core is

$$L_c' = \sqrt{2\tfrac{1}{2}\, D} \tag{27}$$

for even compounding, or

$$L_c'' = \sqrt{3{\cdot}6\, D} \tag{28}$$

for heavy over-compounding.

The cross-sectional area of the yoke-ring may be determined by means of the formula

$$Y_s = \frac{0{\cdot}6\, \lambda\, L_p\, D^{1\frac{1}{2}}\, v}{P} \tag{29}$$

or

$$Y_i = \frac{1{\cdot}5\, \lambda\, L_p\, D^{1\frac{1}{2}}\, v}{P} \tag{30}$$

the subscripts s and i indicating cast-steel and cast-iron, as before.

Having completed the preliminary dimensional calculations for a given size of machine, the ampere-turns per pole may be roughly approximated for a shunt-wound machine by means of the formula

$$A\,t = 100\left[\sqrt{D}\left(\frac{1{\cdot}1\,\sqrt{D}+2}{P} + 1{\cdot}8\,\sqrt{D} + 1\tfrac{1}{2}\right) + 4 - 0{\cdot}6\frac{s}{P}\right] \tag{31}$$

This formula is, of course, based on the assumption of densities and dimensional relations made in the course of this paper, and if those assumptions are carried out, the for-

mula will usually give an excitation value somewhat higher than will be actually required.

In conclusion the author wishes to emphasize the fact that the paper is intended merely to describe the plan of designing that is outlined herein, and not to set up the various assumed values and relations as being in any way superior to those which it may be the practice of other designers to use. It is a simple matter to substitute for the assumed relations used by the author any others that one may prefer, without detriment to the method described, and the latter is submitted in the hope that it may be considered a slight advance over the practice commonly followed in working out direct-current machine designs.

SUMMARY.

Definitions of Symbols:

B_a = Flux density in armature core body.
B_c = Density of commutating magnetic field.
B_d = Density of counter field set up by armature.
B_m = Density of flux in field magnet core.
B_p = Average flux density over pole-face.
B_t = Flux density at roots of armature teeth, assuming no spreading at pole-tips and no flux in armature air spaces.
b = Thickness of a carbon brush set radially.
C = Number of armature coils and commutator segments.
c = Maximum number of segments simultaneously touched by one brush.
D = Outer diameter of armature core.
D_c = Diameter of commutator barrel.
d = Radial depth of armature core between slots and center hole.
E = E.M.F. generated in an armature.
e_c = E.M.F. generated in one coil during commutation.
e_x = Reactance E.M.F. of one coil during commutation.
g = Length of principal air-gap, iron to iron.
g = Average length of air-gap, iron to iron, under an eccentric pole-face.

g_t = Minimum length of air-gap under extreme pole-tip edges.
h = Width of armature core-slot.
i = Current per armature conductor.
j = Width of an armature core-tooth at the root.
K_c = Constant of commutation criterion.
L_a = Gross axial length of armature core.
l = Net axial measurement of iron in armature core.
L_c = Length of a field magnet coil along the core.
L_c' = Total length of coil space for compound winding.
L_p = Axial length of pole-face.
λ = Average pole-face arc ÷ pole-pitch.
M_i = Cross-section of a cast-iron magnet core.
M_s = Cross-section of a cast-steel magnet core.
M_w = Cross-section of a wrought-iron magnet core.
n = Number of armature conductors side by side in one slot.
P = Number of field magnet poles.
p = Number of paths through armature winding.
q = Width or thickness of a rectangular conductor, bare, in mils.
S = Number of coil-slots in armature core.
s = Depth of each coil-slot.
t = Number of turns per commutator segment (or coil).
u = Depth of a rectangular conductor, bare, in mils.
v = Magnetic leakage factor.
W = Watts.
w = Total number of armature conductors = $2\,t\,C$.
Y_i = Cross-section of a cast-iron magnet yoke.
Y_s = Cross-section of a cast-steel magnet yoke.

Assumed relations:

$$B_a = 68{,}000 -$$

$$B_p = 34{,}380\ D^{\frac{1}{6}}$$

$$B_p\,g = 618.8\ D$$

$$B_t = 150{,}000 \text{ to } 160{,}000$$

$$g = 0{\cdot}018\ D^{\frac{5}{6}}$$

TABLE I.

Diameter.	$D^{0.1667}$	$D^{0.667}$	$B_p = 34{,}380\ D^{0.1667}$	$\frac{5000}{D^{0.667}}$	Nearest Rational r. p. m.	Surface Velocity, Ft. per minute.
8	1·414	4·	48,650	1250	1250	2618
9	1·442	4·3267	49,575	1155	1150	2710
10	1·467	4·6416	50,430	1077	1075	2814
11	1·491	4·9461	51,280	1011	1010	2908
12	1·513	5·2415	52,020	954	955	3000
13	1·533	5·5288	52,720	904	900	3063
14	1·553	5·8088	53,390	861	860	3152
15	1·57	6·0822	53,980	822	820	3220
16	1·587	6·3496	54.580	787	785	3288
17	1·604	6·6115	55,140	756	755	3360
18	1·619	6·8683	55,660	728	730	3440
19	1·634	7·1204	56,180	702	700	3482
20	1·648	7·3681	56,660	678	680	3560
21	1·661	7·6117	57,100	657	655	3600
22	1·674	7·8514	57,550	637	635	3657
23	1·686	8·0876	57,980	618	620	3733
24	1·699	8·3203	58,410	601	600	3770
25	1·71	8·5499	58,790	585	585	3830
26	1·721	8·7764	59,170	569	570	3880
27	1·732	9·	59,540	555	555	3923
28	1·742	9·2209	59,850	542	540	3958
29	1·753	9·4391	60,270	530	530	4025
30	1·763	9·6549	60,610	518	520	4083
32	1·782	10·0794	61,270	496	500	4190
34	1·8	10·4951	61,880	476	475	4229
36	1·817	10·9027	62,470	459	460	4335
38	1·834	11·303	63,050	442	440	4377
40	1·849	11·696	63,570	427	425	4450
42	1·864	12·083	64,080	414	415	4563
44	1·879	12·463	64,600	401	400	4607
46	1·893	12·838	65,080	389	390	4697
48	1·906	13·208	65,540	378	380	4775
50	1·919	13·572	65,990	368	370	4843
52	1·932	13·932	66,420	359	360	4900
54	1·944	14·287	66,830	350	350	4947
56	1·956	14·637	67,240	342	340	4984
58	1·967	14·984	67,640	333	330	5010
60	1·978	15·326	68,020	326	325	5105
62	1·989	15·665	68,400	319	320	5194
64	2.	16.	68,760	312	310	5194
66	2·01	16·332	69,100	306	305	5270
68	2·02	16·66	69,445	300	300	5340
70	2·03	16·985	69,790	294	295	5406
72	2·04	17·307	70,130	289	290	5466
74	2·049	17·626	70,460	284	285	5570
76	2·058	17·942	70,750	279	280	5570
78	2·067	18·256	71,060	274	275	5615
80	2·075	18·566	71,360	269	270	5654

$$i\,w\,\lambda = \frac{B_p\,P\,g}{2{\cdot}532}$$

$$l = 0{\cdot}8\,L_p$$

$$q\,u = 500\,i$$

$$r.\,p.\,m. = 5000 \div D^{\frac{3}{2}}$$

$$s = 1 + 0{\cdot}125\,\sqrt{D}$$

NOTE.—All dimensions are expressed in inch measurements, except the size of armature conductors, which is in mils.

TABLE II.

RELATIONS BETWEEN POLEFACE DENSITY AND MAGNETIC PATH IN ROOTS OF ARMATURE TEETH.

Numbers in the body of the table are values of $\frac{Sj}{D + 2g}$.

$\frac{l}{L_p}$	$B_t \div B_p$						
	2	2·1	2·2	2·3	2·4	2·5	2·6
0·80	1·96	1·87	1·78	1·71	1·64	1·57	1·51
0·81	1·94	1·85	1·76	1·69	1·62	1·55	1·49
0·82	1·92	1·82	1·74	1·67	1·60	1·53	1·47
0·83	1·89	1·80	1·72	1·65	1·58	1·51	1·46
0·84	1·87	1·78	1·70	1·63	1·56	1·50	1·44
0·85	1·85	1·76	1·68	1·61	1·54	1·48	1·42
0·86	1·83	1·74	1·66	1·59	1·52	1·46	1·40
0·87	1·81	1·72	1·64	1·57	1·50	1·44	1·39
0.88	1·78	1·70	1·62	1·55	1·49	1·43	1·37
0·89	1·77	1·68	1·60	1·53	1·47	1·41	1·36
0·90	1·75	1·66	1·59	1·52	1·45	1·40	1·34
0·91	1·73	1·64	1·57	1·50	1·44	1·38	1·33
0·92	1·71	1·63	1·55	1·48	1·42	1·37	1·31
0·93	1·69	1·61	1·54	1·47	1·41	1·35	1·30
0·94	1·67	1·59	1·52	1·45	1·39	1·34	1·29
0·95	1·65	1·57	1·50	1·44	1·38	1·32	1·27

TABLE II-a.

RELATIONS BETWEEN POLEFACE DENSITY AND MAGNETIC PATH IN ROOTS OF ARMATURE TEETH.

Numbers in the body of the table are values of $\frac{Sj}{D + 2g}$

$\frac{l}{L_p}$	$B_t \div B_p$						
	2·7	2·8	2·9	3	3·1	3·2	3·3
0·80	1·45	1·40	1·35	1·31	1·27	1·23	1·19
0·81	1·44	1·39	1·34	1·29	1·25	1·21	1·18
0.82	1·42	1·37	1·32	1·28	1·24	1·20	1·16
0·83	1·40	1·35	1·31	1·26	1·22	1·18	1·15
0·84	1·39	1·34	1·29	1·25	1·21	1·17	1·13
0·85	1·37	1·32	1·27	1·23	1·19	1·16	1·12
0·86	1·35	1·30	1·26	1·22	1·18	1·14	1·11
0·87	1·34	1·29	1·25	1.20	1·17	1·13	1·09
0 88	1·32	1·28	1·23	1·19	1·15	1·12	1·08
0 89	1·31	1·26	1.22	1·18	1.14	1·10	1·07
0·90	1·29	1.25	1·20	1.16	1·13	1·09	1·06
0·91	1.28	1.23	1·19	1·15	1·11	1·08	1·05
0·92	1·26	1·22	1·18	1·14	1·10	1·07	1·03
0·93	1·25	1·21	1·16	1·13	1·09	1·06	1·02
0·94	1·24	1·19	1·15	1·11	1·08	1·04	1·01
0·95	1·22	1·18	1·14	1·10	1·07	1·03	1·00

A NEW USE FOR ALUMINUM.

Bernhard, a German experimenter, noting the structure of aluminum, concluded to try it for putting an edge on fine cutting instruments such as surgical knives, razors, etc. To his surprise and pleasure, it acted exactly like a razor hone of the finest quality. Further investigation showed that the metal had other characteristics that made it pre-eminently useful in this direction. When steel is rubbed on it, as for instance, in honing a knife blade, the metal disintegrates, forming an infinitely minute powder of a greasy unctuous feeling that clings to steel with great tenacity, and thus assists in cutting away the surface of the harder metal. So fine is the edge produced that it can in no wise be made finer by the strop, which, used in the ordinary way, merely tends to round the edge. The writer of the paragraph has confirmed most of Bernhard's statements, though he believes he can get an edge with the Ouachita (Hot Springs) hone equally as fine as aluminum gives, and equally as rapidly.—*National Druggist.*

ELECTRICAL SECTION.

Stated Meeting held May 28, 1903.

Static Discharges in Electric Circuits.

By Percy H. Thomas.

INTRODUCTORY.

"Static discharges" in electric circuits supply a very interesting and instructive example of one of the most universal of nature's methods—perhaps the most general of her ways of transferring energy, viz., wave motion. Wave motion plays a very prominent part in nearly every department of physics: heat, light, sound, electricity, dynamics. In its broadest application, wave motion serves to accomplish the transfer of energy. Although in the different branches of physics in connection with which wave motion may be studied, different sorts of forces are involved, and different kinds of effects are produced, yet, at least as regards the wave motion itself, the laws of transmission, form, etc., are the same in all.

In electric circuits, the presence of waves and their form is often extremely difficult to examine on account of troubles in measurement, the great speed with which electrical waves travel, etc. In other departments of physics, however—for example, motion in liquids—the study of waves is comparatively easily accomplished, as their form and progressive changes are visible to the eye. There is a great advantage, therefore, in analyzing electric waves to as far as possible reason by analogy from the more familiar and easily comprehended forms of wave motion, *e. g.*, water waves.

It is an unfortunate fact that, although the formation and progress of waves appear very simple and easily appreciated, yet the mathematical treatment of the subject is very difficult and much involved.

It is the purpose of the present paper to generalize a little on the character of wave motion in general, including the

simplest and most important laws involved, and then treat by analogy the effects of waves in electric circuits. It will then be proper to treat the practical bearing of the wave phenomena developed in commercial electric circuits and the available methods of avoiding any injurious effects that might be anticipated from such waves.

Wave Motion in General.—As has already been stated, the general result accomplished by wave motion is the transfer of energy from place to place with more or less change of its form. In the same way as matter, energy may be said to exist; that is, may be considered to be a transferable entity. Energy exists in definite quantity at a definite point and may be transferred from this point to another. Energy, which is found in well-known sources, such as fuel, water-power, radiant energy from the sun, kinetic energy of the solar system, etc., is of commercial use only when existing in an available form at the particular point where useful work is to be done. The transportation of energy is consequently of the greatest importance.

Mathematically speaking, energy or work is developed when motion is produced against a resistance, and the amount of work or energy developed equals the product of the distance of motion by the amount of resistance.

It is evidently possible to transfer energy in two general ways:

(1) By transferring the matter with which energy may be associated, and

(2) By causing the energy to develop a force and causing this force to develop another force at the point to which the energy is to be transferred. The latter method of transfer is the one involved in wave motion.

But there are two ways of energy transfer in the method, involving the transfer of force as force. The first requires a rigid connection between source and receiving point, by which any motion at the source of energy causes a simultaneous and equivalent motion at the receiving point; such, for instance, is the connecting rod of an engine. In the first case the force at the source is at all instants of time equivalent to the force at the receiving point.

In the second case inertia and elasticity are necessarily involved in the transfer of the force; the motion at the source causes motion in its immediate neighborhood, slightly later in time on account of the elasticity assumed, and this motion in turn causes motion slightly further from the source a little later in time and so on until the receiving point is reached, perhaps considerably after the development of the original force. This type of transfer of energy is illustrated by the passage of a wave over a surface of water. Here the original development of force, represented by the elevation of the water at the crest of the wave, evidently passes from one point to another, each point receiving the motion at a slightly later instant of time than the previous point. This second case is, strictly, wave motion, and, as already stated, assumes or requires the presence or the equivalent both of elasticity and inertia in the medium by which the force is transferred from source to receiving point.

Both the latter forms of energy transfer require a continuous medium between the source and the receiving point; in the first case the medium cannot have both elasticity and inertia. In the first case the result at the receiving point is coincident in time with that at the source. In the second, the effect at the source is at a later time. The same transfer of energy may be accomplished by either method through its proper medium.

By both methods the exact rate of development of energy will be reproduced at the receiving point except for an absorption of energy in the medium as the force is transferred through it.

Some examples of the two types of transformation will be interesting. Of the first form, the transfer through a rigid beam like the connecting rod of an engine has been mentioned. Belt, rope or gear transmission is another example. Also the transfer of energy from pump to work piston in a hydraulic pressure apparatus. In electricity, an electric circuit containing a generator and motor, provided the circuit contain no electrostatic capacity, is another example. Here the current flowing produces an exactly equivalent and simultaneous force in the generator and

motor. In the second case, as examples, there are the transfer of sound, of light, of heat, of water waves, and static waves in electric circuits containing comparatively large inductance and capacity. In such circuits evidently the first flow of current passes to the capacity near the source of current, and at later periods of time current appears at points more remote from the source and last of all at the receiving point; that is, there is an equivalent force generated both at the generator and at the motor (provided no material loss occurs in the rest of the circuit) but the time at which this force appears at the motor is later than that at which it first appears at the generator. Telephony gives another and very perfect example of electric wave motion.

It will not be necessary to refer again more than casually to the transfer of energy through a rigid medium which represents the ordinary form of commercial transfer of power in an electric circuit.

Nature of Wave Motion.—To develop a little more carefully the nature of wave motion, it will be well to consider another example of energy transfer of the second class, that is through an elastic medium possessing mass, viz., the starting of a long train of heavily loaded cars by a locomotive. In this case the locomotive is the source of energy, the last car may be assumed to be the receiving point; the springs in the draw bars of the cars are the elasticity of the medium, and the weight of the cars its inertia. When the locomotive starts forward the first car starts first, and then as the spring between it and the next extends, the second car starts, extending the second spring and causing the third car in turn to start, etc., the first motion of the locomotive being thus transferred down the train to the end. Here, the time-lag between the development of force at the source and at the receiving point is very evident. It is evident, also, that this lag is due to the combination of the inertia and elasticity in the train. If the locomotive be reversed when the train is all in motion, the first car, as before, will receive backward motion first, the second, through the first spring, a little later, and so on to the end of the train, the last car being the last to feel the reverse impulse. This

starting impulse which passes from the front to the rear of the train is really a wave and is transferred through to the medium in exactly the same way that a sound is transferred through a tube. It must be noted that the first impulse of the locomotive started the first car and this the second, and so on, and that the original impulse would pass the length of the train to the end whether the locomotive, after delivering its push, stopped immediately or continued in its motion. If the second impulse of the locomotive was to stop then after delivering the first starting impulse to the second car, the first car would be stopped by the locomotive, it in its turn would stop the second and so on, the second impulse following the first down the train to the end. If the second impulse of the locomotive had been another impulse forward, the first car would receive a second impulse forward, followed by the second car, etc., and the last car would receive the two impulses as separately as the first car or the second. The same length of time would elapse between the first and second impulses at the last car as at the locomotive.

If all the cars be of the same weight and the springs of the same strength, all waves of motion produced by the locomotive will pass down the train with the same velocity. This fact is generally true in wave transfer. All waves tend to pass at a uniform rate through the mediums which have the same elasticity and inertia characteristics. Mathematically speaking, the velocity of motion of the wave forward is proportional, inversely, to the square root of the product of the elasticity and the inertia. In cases where there is a loss of energy as the wave passes through the medium, a certain slight alteration in the velocity results.

Returning to the example of the train; if we assume that a locomotive starts forward and continues its motion for some time until the whole train has attained the same velocity, and then stops and runs backward for the same distance, and repeats this cycle again, we have an alternating force, so to speak, and waves alternating in direction are sent down the train, but each forward wave is completed before a negative wave begins, and vice versa. Sup-

pose, on the other hand, however, that the locomotive starts forward and then backward rapidly so that the backward movement of the locomotive is accomplished before the previous forward movement has reached the rear of the train; we then still have the alternating force at the locomotive and a constant series of waves passing down the train, some positive and some negative. That is, cars in some parts of the train are moving forward and others moving backward all at the same instant.

In the latter case the distance along the train between the two nearest cars which are moving forward at the same time is called the wave length. It is the distance which the original motion will travel during the time required for the locomotive to have completed one cycle of movements and to begin repeating this cycle. Wave length in water waves is the distance between the crests of two adjacent waves, which is, of course, the same as the distance between the troughs. The wave length evidently depends upon two factors: (1) the speed with which the wave travels in the medium, and (2) the time required for the exciting force to complete a cycle movement. The term wave length is used chiefly where the exciting source goes through regular repetitions of the cyclic motion.

In the first case above in which the locomotive required a long time for completing a cycle of motion (that is going forward and returning) the effect on the rear car was *practically*, though not exactly, the same as though the train had been perfectly rigid and the transfer of energy had been made after the same fashion as that of a connecting rod. In the second case, however, in which the alternations of motion of the locomotive forward and backward were so rapid that several occurred before the end of the train was affected, the motion of the rear car, though of the same nature exactly as that of the locomotive, was yet very different as regards the time at which such motion occurred relatively to the time of the exciting motion in the locomotive; that is, there is a great time lag between the source and the receiving point. This time lag is of more importance in electric circuits than in the case cited.

Recapitulation.—When a force is produced at a source of energy at one end of a proposed line of transmission for this energy, a wave of force is started which passes along the line at a constant, definite rate (which is inversely proportional to the square root of the product of the elasticity and the inertia of the medium in which the motion exists) until the end of the line is reached. If the original force be alternating in its nature, a succession of waves of opposite directions will follow one another to the receiving end, keeping a constant distance apart, equal to the space traversed by a wave during the time of one cycle of the source. If this cycle be slow in comparison with the time required for a wave to traverse the whole line, the effect at the receiving end will be practically the same as though the force were directly applied at this point. This action is closely analagous to the transmission of power in ordinary commercial circuits in which there is only a comparatively small capacity connected with the system which delays the current only slightly as it starts up, and which allows the action both at the generator and motor end to be substantially simultaneous. In the case where the cycles of the source are very quick in comparison with the time required for the way to pass the length of the line, however, there are a number of waves passing down the line at the same time, and the shortest distance between two similar waves is called the wave length. The wave length depends upon the velocity of the wave in the medium and on the frequency of the alternations of the source. In any given circuit different wave lengths can be obtained by applying alternating forces of different frequency. In commercial circuits since the velocity of the transmission of electric waves in straight wires in the air is approximately the velocity of light, the only common method by which sufficiently rapid alternations of applied force can be obtained to cause several waves in a line at one time is by a static discharge; that is, a sudden changing of the potential of a portion of the line caused by allowing it to discharge over an air-gap or its equivalent.

Laws of Wave Motion.—As supplementary to this descrip-

tion of the nature of wave motion, the simplest laws governing such motion will be discussed. After a general treatment of these laws their application to commercial electric circuits will be considered.

Suppose, in the first place, the source of energy is in the center of a large homogeneous medium capable of transmitting the wave in any direction, for instance; a tuning fork in the center of a large room. Waves will be sent out in all directions with each motion of the tuning fork. It is evident that as each wave passes farther and farther from the source, since its velocity is the same, a larger and larger quantity of medium must be affected. This means, evidently, that the intensity of the wave at any particular point of the medium gets less and less as it passes farther and farther from the source. Since the amount of the medium affected at any distance from the source varies as the square of this distance, the intensity of the wave varies inversely as the square of its distance from the source. This is the well-known inverse square law governing the intensity of sound, light, heat and of any form of energy which can be radiated from a point. This is not the case involved in electric circuits, however, and will not be further discussed.

If, instead of being placed in a medium in which they can pass in all directions, the waves are confined to one plane, as the waves on the surface of water, waves will again be transmitted in all directions in this plane and will again diminish in intensity, since larger and larger amounts of the medium are involved as the waves get further from the source, but in this case the diminution is only inversely as the first power of the distance, since the amount of the medium excited by the wave in a given time increases only in this ratio.

Suppose again, however, that the medium be confined to a tube, a speaking tube, as for sound, and be allowed to pass in one direction only. In this case, since waves do not excite any larger portion of the medium at a distance from the source than nearby, there is no diminution of intensity due to the spreading out of the original wave as it

proceeds farther and farther from the source. The electric circuit is analogous to the case of the tube-shaped medium, therefore this form only, called linear waves, will be discussed. There may, however, be a certain loss of energy due to resistance to motion in the medium, or other supplementary waste of energy which is not characteristic of wave motion in a perfect medium. This will tend, even with linear waves, to lessen the energy of the wave and change its form somewhat. The most important laws governing linear waves will be discussed under their appropriate headings.

Velocity.—It has already been stated that the velocity is inversely proportional to the square root of the elasticity times the inertia, and is slightly altered by any waste of energy in the transmission of the wave.

Wave Form.—The general form and amount of energy in the wave are determined at the instant of its creation at the source, and in a perfect medium would remain unchanged through all transmission. Energy losses, however, somewhat modify the form of the wave as well as altering its velocity. Once a wave has left the source, no subsequent action of the source can modify it directly.

Reflection.—When a wave reaches the end of a conducting medium, and can go no farther, it is reflected backward without change except of direction, and returns back through the medium toward the source at the same velocity. The effect is exactly the same as though the original wave went on to infinity and a new wave were started exactly like the first at the reflecting point, at each instant exactly similar to the outgoing wave. Thus, it will be seen at the reflecting point for the instant of reflection there will be twice the intensity of the original wave. If this be an electric wave, there will be twice the potential at this instant. The increased potential exists only during the time of reflection.

If, instead of meeting an obstacle in its path at the end of the line, the wave meets a point where all of its energy can be absorbed, such as a short circuit or a large condenser in an electric circuit or an open mouth of a trough of water.

the wave will still be reflected, but this time with a reversal of the direction of this intensity; that is to say, a wave will start backward from the reflecting point as in the case of meeting the obstacle as above, but will be of a negative sign, a lowering of potential in the electric circuit instead of a raising potential. This will pass on toward the source as before.

In acoustics the former case represents a closed organ pipe in which the sound is reflected at the closed end and returns as a condensation toward the lip of the pipe. The second case illustrates the open organ pipe in which a rarefaction is sent back toward the lip by reflection.

If the reflected wave reach the generator it will there be reflected into the line again with reversed sign without altering any other waves which the generator may be sending out.

Nodes and Loops.—It is evident from the paragraph above that if a continuous series of alternating waves be produced

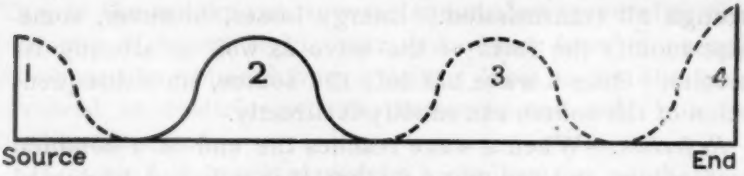


FIG. 1.—Passage of wave.

by the source and these be reflected from the receiving end, whether it be an obstacle (an open wire), or short circuit, the returning reflected waves and the new series of outgoing waves will pass one another. The resultant effect will be the sum of the two; that is, where a wave returning meets a wave going out, there will be a wave of double height for the moment. Where a wave returning meets a wave going out of the opposite sign, however, these two will neutralize and there will be no effect at that particular point. A little consideration will show that the returning positive wave will meet outgoing positive waves always at the same points and that the returning positives will meet the outgoing negatives, and vice versa, always at the same points. Consequently, the points of extra high intensity and points of

zero intensity are fixed, the natural result being that a series of high and low points are kept reappearing and disappearing as the waves pass out and back, but which are stationary in location. These are called nodes and loops, and the highest points are double the intensity of the individual waves sent out. The outgoing waves and the nodes and loops are illustrated in *Figs. 1* and *2*. This double voltage is really the same phenomenon as the double voltage produced by reflection at the closed end of the wire or tube.

Resonance.—When a returning wave reaches the generator it is reflected into the line again with change of sign as

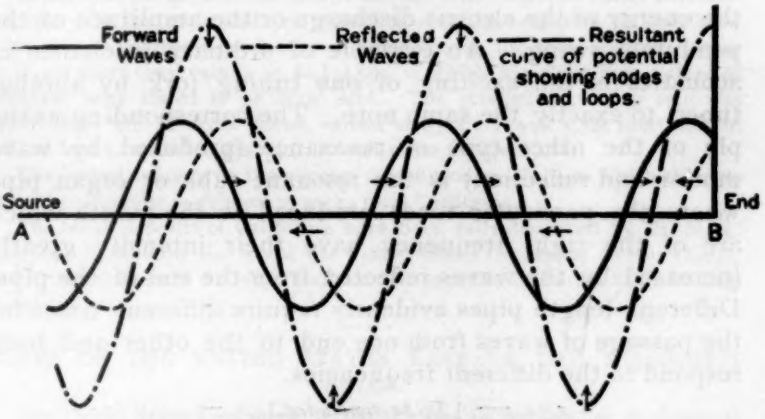


FIG. 2.—Nodes and loops in a long line or cable.

already described. If at the same time the generator be sending out a wave of the same sign as the reflected wave, a wave of double intensity will proceed to the end of the line, be reflected and return to the generator, where it will again be reflected with reversed sign, and the new wave which will just be leaving the generator, with the double wave just reflected by the generator, will form a wave of triple magnitude. This will return to the source as before, and step by step intense waves will be built up which will theoretically reach an infinite value, provided no loss of energy occurs; but such losses do occur in all cases, which fact allows the intensity of the wave to build up only to such a

point that this energy loss equals the energy supplied by each additional wave. This phenomena is perfectly analagous to resonance and is, in fact, a more general case of resonance. Ordinarily it is considered that resonance occurs when inductance and capacity occur in a circuit in such proportions that the condenser discharging through the inductance has its charge oscillate from one side to the other of the condenser as a pendulum oscillates and in addition there be applied in the oscillating circuit an alternating electromotive force, or in the case of the pendulum an alternating force, which has exactly such a frequency of alternation that the impulse of the electromotive force, or the force, as the case may be, is just right to increase the energy of the electric discharge or the amplitude of the pendulum swing. An example of ordinary resonance in acoustics is the exciting of one tuning fork by another tuned to exactly the same note. The corresponding example of the other type of resonance (produced by wave motion and reflection) is the resonant tube or organ pipe where the particular tones produced at the mouth which are of the right frequency have their intensity greatly increased by the waves reflected from the end of the pipe. Different length pipes evidently require different times for the passage of waves from one end to the other and back respond to the different frequencies.

[*To be concluded.*]

Notes and Comments.

PRODUCTION OF ASPHALTUM AND BITUMINOUS ROCK IN 1902.

The production of asphaltum and bituminous rock in 1902, according to the report of Dr. Joseph Struthers to the United States Geological Survey, now in press, showed a large increase over that of 1901, amounting in quantity to 36,525 short tons and in value to $122,259, the figures for the two years being, respectively, 99,659 short tons ($677,594) and 63,134 short tons ($555,335). The relatively smaller increase in value as compared with quantity was due to the very large proportion of bituminous sandstone produced.

The production of hard and refined asphaltum increased from 19,316 short tons in 1901 to 22,321 short tons in 1902. The production of liquid asphaltum, all of which was derived from California, decreased from 2,600 short tons in

1901 to 1,605 short tons in 1902. The quantity of asphaltum produced in the refining of crude oil during 1902 amounted to 16,027 short tons.

The imports in 1902 amounted to 153,093 long tons ($492,604), as compared with 138,833 long tons ($553,473) in 1901.

The report contains a brief abstract of the production of asphaltum in Cuba in 1901.

PRODUCTION OF BORAX IN 1902.

The known deposits of borax in the United States, according to the report of Dr. Joseph Struthers to the United States Geological Survey, are in California, Nevada and Oregon, and the chief producer of borax and boric acid in this country is the Pacific Coast Borax Company, which owns the plant at Daggett, San Bernardino County, Cal., where the colemanite (calcium borate) is bedded in deposits from 5 to 30 feet thick. This company acquired much additional property during 1902 in the Death Valley region.

It has not been practicable to separate the total output of borax salts during 1902 into the crude and the refined product; the reported returns, however, give an aggregate production of refined borax and boric acid amounting to 17,202 short tons, valued at $2,434,999, of which 862 short tons, valued at $155,000, were stated to be boric acid. The production during 1901 was 17,887 short tons of crude borax, valued at $314,811, and 5,344 short tons of refined borax, valued at $697,307, a total value of $1,012,118.

The largest refinery in the country is at Bayonne, N. J., to which the pulverized colemanite is shipped in sacks from California.

The borax deposits of California have been fully described by Mr. M. R. Campbell in Bulletin No. 200 of the United States Geological Survey (1902), and by Mr. Gilbert E. Bailey in Bulletin No. 24 of the California State Mining Bureau (1902).

CLAYS OF THE UNITED STATES EAST OF THE MISSISSIPPI RIVER.

The United States Geological Survey is about to publish, as Professional Paper No. 11, "The Clays of the United States East of the Mississippi River," by Dr. Heinrich Ries, a paper that has been looked for with interest by a large number of persons, particularly by those engaged in some form of the clay industry.

Dr. Ries discusses briefly the origin, physical and chemical properties, methods of mining, purification, commercial value, and uses of clay, and the geologic distribution of clays east of the Mississippi according to their rock derivation; and then he takes up the distribution of clays by kinds, and the description of clay deposits by States. He concludes with a summary of the clay-working industry east of the Mississippi, touching in turn and by States on the manufacture of common brick, pressed brick, fire-proofing, roofing tiles, terra cotta, enameled brick, floor tiles, glazed tiles, vitrified brick, drain tile, sewer pipe, fire brick, and pottery.

The largest brick-making region in the country is the Hudson River Valley, in New York State, where nearly a billion brick are made annually. Pennsylvania leads in the production of pressed brick. Most of the terra

cotta comes from New York, New Jersey, and Illinois. Although West Virginia was the cradle of the paving-brick industry, Ohio now leads in the production of vitrified brick. Ohio, Illinois, Indiana, and Michigan are the most important producers of drain tile, and Ohio is the main producer also of sewer pipe. Pennsylvania produced over $4,500,000 worth of fire brick, about one-half the total production, in both 1900 and 1901. Ohio, New Jersey, and Pennsylvania, in the order named, are the greatest producers of pottery, East Liverpool, O., and Trenton, N. J., being the chief centers of production.

Franklin Institute.

[*Proceedings of the stated meeting held Wednesday, October 21, 1903.*]

HALL OF THE FRANKLIN INSTITUTE,
PHILADELPHIA, October 21, 1903.

Vice-President JOHN BIRKINBINE in the chair.

Present, 162 members and visitors.

Additions to membership since last meeting, 14.

Mr. J. B. Fontaine, Jr., presented a model of a working-beam engine made by the late Mr. Henry Cartwright, a former vice-president of the Institute. A vote of thanks was tendered to the donor.

Mr. John W. Hill, Chief Engineer, Department of Public Works, gave a description of the Belmont Filtration Plant now in course of construction by the city of Philadelphia. Mr. Hill's communication was profusely illustrated by means of lantern photographs. The president extended the thanks of the meeting to the speaker for his able and exhaustive contribution to the subject.

Adjourned,

WM. H. WAHL,
Secretary.

Committee on Science and the Arts.

[*Abstract of proceedings of the stated meeting held Wednesday, Oct. 7, 1903.*]

MR. CHAS. E. RONALDSON in the chair.

The following reports were adopted:

(No. 2202.) *Exposure Scale for Timing Negatives.*—Edward Wagner-Smith, Philadelphia. (An advisory report.)

(No. 2237.) *Tube Expander.*—Chas. V. Burton, London, England. (An advisory report.)

The following reports passed first reading:

(No. 2164.) *Steam Generator.*—John C. Parker, San Francisco, Cal.

(No. 2303.) *Measuring Device.*—Edgar Hill, Philadelphia.

These were held under advisement.

Protests against reports were received in the following cases:

(No. 2268.) Rulon's *Electric Battery.*

(No. 2272.) Eberhardt's *Radial Gang Cutter.*

These were referred to sub-committees.

W.